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Endogenous participation in charity auctions

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ABSTRACT

Data from a recent field experiment suggests that differences in participation rates are responsible for much of the variations in charity auction revenues across formats. We provide a theoretical framework for the analysis of this and other related results. The model illustrates the limits of previous results that assume full participation and introduces some new considerations to the choice of auction mechanism. It also implies, however, that the data cannot be explained in terms of participation costs alone: there must exist mechanism-specific obstacles to participation.

1. Introduction

From the small town silent auction that raises a few hundred dollars to the $70 million Robin Hood annual benefit in New York City (Anderson, 2007), charities and non-profits often use auctions to transform donations in kind into cash. The choice of format constitutes a difficult decision problem, however, even under idealized circumstances: if all bidders, win or lose, derive some benefit from monies raised, revenue equivalence does not hold, even if valuations of the object itself are private and independent. [1] It is only in the last few years, however, that a small but vibrant literature on the economics of charity auctions has developed.[2]

The best known theoretical finding is perhaps Goeree et al.'s (2005) result that when the standard (SIPV, or single object, independent private values) auction with risk neutral bidders is extended so that all bidders also receive some revenue proportional benefit, all-pay auctions produce more revenue than any winner-pay auction. The intuition, as they characterize it, is that winner-pay mechanisms suppress bids because when one bidder tops the others, she wins the object but loses the chance to free ride on the benefits associated with the best of the other bids. While there are few, if any, examples of all-pay auctions, the result seems to rationalize the widespread use of raffles and lotteries, both of which could be viewed as practical variations on the all-pay theme. Engers and McManus (2007) have since shown that if bidders who contribute experience an additional "warm glow" (Andreoni, 1995), the superiority of the all-pay over both first-price and second-price winner pay mechanisms survives in the limit, as the number of bidders increases.

Two recent lab experiments would seem to support these results. Davis et al. (2006) find that lotteries raise more revenue than English auctions, while Schram and Onderstal (forthcoming) conclude that lotteries do worse than all-pay auctions but better than first price auctions. On the other hand, Carpenter et al. (2008), who conduct one of the few field experiments on charity auctions, reach a quite different conclusion, namely, that the all-pay mechanism generates no more revenue, in a statistical sense, than the second price sealed bid, and that both generate less revenue than the familiar first price sealed bid. The difference is the result of endogenous participation: the model in Goeree et al. (2005) and the experimental designs in Schram and Onderstal (forthcoming) and Davis et al. (2006) all assume a fixed number of bidders but Carpenter et al. (2008) found that in the field, the ratio of active to potential bidders, or participation rate, was much lower under all-pay rules.

These experimental results prompt an important question: what is the theoretical relationship between participation costs, understood here in the broadest sense of the word, and revenue in auctions with proportional benefits? Our purpose in this paper is to describe and then characterize a model of endogenous participation that allows for mechanism-specific entry costs.

The next section reports, in the form of a pair of propositions (the proofs of which are available as an online appendix), the optimal symmetric bid functions and expected revenue functions for the first price, second price and all-pay sealed bid SIPV auctions in which *all* bidders, active or otherwise, earn a benefit that is proportional to

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[1] This characteristic is not unique to charity auctions: Engelbrecht-Wiggans (1994) counts Amish estate sales and buyer ring knockouts as examples of auctions with what he calls "price proportional benefits."

[2] In addition to the contributions of Goeree et al (2005), Engers and McManus (2007), Davis et al. (2006) and Schram and Onderstal (forthcoming), which we discuss in more detail, other recent examples include Elfenbein and McManus (2007), Leszczync and Rothkopf (2007) and Isaac et al. (2007).

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revenue, and those who contribute to the charity (the winner in first and second price auctions, but all active bidders in all-pay auctions) experience a warm glow proportional to their bids, under the assumption that the submission of a bid imposes some cost on bidders. This representation of the participation problem owes much to the recent work of Menezes and Monteiro (2000) and, much earlier, Samuelson (1985).

In the third section, we explore the properties of these bid and revenue functions, both within and across mechanisms, a more complicated task than first seems. We know from the work of Menezes and Monteiro (2000), for example, that in the absence of revenue proportional benefits, revenue equivalence is preserved under the introduction of participation costs, but that this (common) revenue function can exhibit some unusual properties. It need not be the case, for example, that expected revenue rises with the number of potential bidders, or that in the limit, it is independent of the distribution of private values. On the other hand, we learn from Engers and McManus (2007) that even without participation costs, there is no fixed order of revenues in small (low N) auctions with revenue proportional benefits and warm glow. To cultivate a sense of what properties do, and do not, prevail in practice, we calculate and plot numerical bid and revenue functions for several members of the Kumaraswamy (1980) family of bounded value distributions. In the process, we consider what costs, and cost differentials, would be consistent with the experimental literature.

The fourth section considers the relationship of this model to previous empirical work, and we conclude with a brief discussion of possible future research.

2. Optimal bids and expected revenues

Our model starts with $N \geq 2$ potential risk neutral bidders whose private values for some indivisible object can be modeled as independent draws from some continuously differentiable distribution function F over the unit interval $[0,1]$. These values are known to bidders *before* the decision to participate (or not) must be made. Auction revenues are used to provide a service from which all bidders, active or inactive, benefit. As in Goeree et al. (2005), the value to each bidder is a constant fraction $0 \leq \alpha < 1$ of these revenues. Some active bidders will also experience a "warm glow" (Andreoni, 1995; Engers and McManus 2006) equal to a fraction $0 \leq \gamma < 1-\alpha$ of their own contribution to auction revenue. The limit on γ is needed to ensure that each bidder's optimization problem is well-defined, implies that $\beta = \alpha + \gamma$, the sum of the common return and warm glow, is also less than one.

Following Samuelson (1985) and Menezes and Monteiro (2000), potential bidders face some cost of participation $0 \leq c^j < 1$, $j = f(irst\ price), s(econd\ price), a(ll\ pay)$, the value of which could be mechanism-specific. As a result, the number of *active* bidders is not predetermined. Samuelson (1985) defines the cost in terms of the resources committed to "bid preparation" but, on the basis of the previous discussion, our interpretation is somewhat broader and includes, for example, the disutility of participation in an unfair mechanism. While the cost is allowed to vary across mechanisms – a feature we exploit to explore both the effects of participation costs per se and cost differentials on the "traditional" revenue ordering – it is assumed to be the same for all bidders. As we also note in the conclusion, however, this form of bidder asymmetry is an important direction for future research.

Within this framework, the derivation of optimal bid functions draws heavily on both Menezes and Monteiro (2000) and Engers and McManus (2007):

Proposition 1. The Bayes-Nash symmetric bid functions are:

$$\sigma^f(v) = \frac{1}{1-\gamma}\left[v - \frac{F(\underline{v}^f)^\theta}{F(v)^\theta}\underline{v}^f - \frac{1}{F(v)^\theta}\int_{\underline{v}^f}^{v} F(x)^\theta dx\right] \tag{1}$$

$$\sigma^s(v) = \frac{1}{1-\gamma}v + \frac{1}{(1-\gamma)(1-F(v))^{\frac{1-\gamma}{\alpha}}}\int_v^1 (1-F(x))^{\frac{1-\gamma}{\alpha}} dx \tag{2}$$

$$\sigma^a(v) = \frac{1}{1-\beta}\left(vF(v)^{N-1} - \underline{v}^a F(\underline{v}^a)^{N-1}\right) - \frac{1}{1-\beta}\int_{\underline{v}^a}^{v} F(x)^{N-1} dx \tag{3}$$

where $\theta = (1-\gamma)(N-1)/(1-\beta)$, with participation thresholds implicitly defined by:

$$F(\underline{v}^j)^{N-1}\underline{v}^j = c^j \qquad j = f, a \tag{4}$$

$$F(\underline{v}^s)^{N-1}\underline{v}^s + \alpha(N-1)F(\underline{v}^s)^{N-2}(1-F(\underline{v}^s))\sigma^s(\underline{v}^s) = c^s. \tag{5}$$

It is then not difficult to derive the expected revenue functions:

Proposition 2. Given the bid functions (1), (2) and (3), expected revenues are equal to:

$$R^f = N\int_{\underline{v}^f(c^f,N)}^1 F(v)^{N-1} f(v)\sigma^f(v)dv \tag{6}$$

$$R^s = N(N-1)\int_{\underline{v}^s(N,c^s,\alpha,\beta)}^1 F(x)^{N-2}(1-F(x))f(x)\sigma^s(x)dx \tag{7}$$

and:

$$R^a = N\int_{\underline{v}^a(N,c^a)}^1 \sigma^a(v)f(v)dv \tag{8}$$

The proofs of both propositions and other technical material are available in the online appendix.

3. Comparison of mechanisms

3.1. Numerical analysis and the Kumaraswamy distribution

We observed, in the introduction, that there is no fixed order for the three mechanisms under endogenous participation. This does not mean, however, that the mechanisms are without "common properties" that should inform both research and practice. To determine whether such properties exist, we shall compare participation, bid and revenue functions when the distribution of private values over the unit interval is a member of the Kumaraswamy (1980) family:

$$F(v|a,b) = 1-(1-x^a)^b \quad a,b > 0 \tag{9}$$

with mean $b\Gamma\left(1+\frac{1}{a}\right)\Gamma(b)/\Gamma\left(1+\frac{1}{a}+b\right)$.[3] Much of the discussion that follows will focus on the four particular examples with the implied density functions depicted in Fig. 1: $F(v|1,1)$, the standard uniform distribution with mean 0.50, and a benchmark in the literature; $F(v|2,2)$, which has almost the same mean as the uniform distribution (0.53) but is hump-shaped, the equivalent of an auction in which few "extreme bidders" should be expected; $F(v|3,1)$, with mean 0.25, which produces auctions with an expected preponderance of "low value bidders"; and $F(v|1,5)$, with mean 0.83, which instead leads to auctions with a disproportionate number of "high value bidders."

3.2. Threshold values and participation rates

It is an immediate consequence of Proposition 1 that if participation costs in first price and all-pay auctions are the same, the threshold values and rates of participation should be, too. To understand this, we first note that if the "threshold bidder" – that is,

[3] The Kumaraswamy distribution is one of the simplest and most tractable families of "double bounded" distributions.

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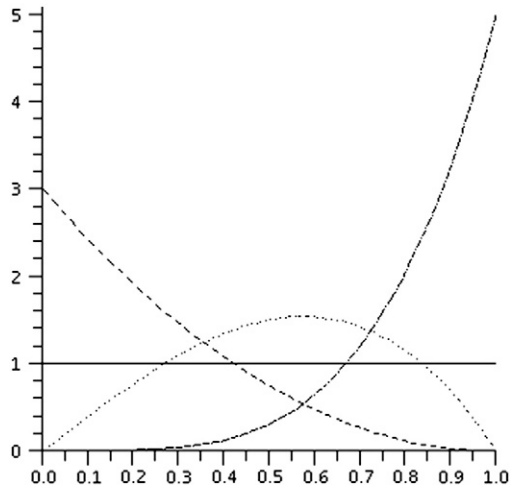

Fig. 1. Kumuraswamy density functions.

the bidder with private value $\underline{v}^j$ – does indeed decide to bid, she should bid zero. With likelihood $F(\underline{v}^j)^{N-1}$, she will, as the lone participant, win the auction and receive benefits equal to her private value $\underline{v}^j$.[4]

With likelihood $1-F(\underline{v}^j)^{N-1}$, she will lose the auction, however, and receive benefits B^j that depend (only) on the actions of other bidders. It follows, then, that the net benefits of participation are $F\left(\underline{v}^j\right)^{N-1}\underline{v}^j + \left(1-F\left(\underline{v}^j\right)^{N-1}\right)B^j - c^j$. Because the benefits B^j are not limited to participants, however, she receives a benefit equal to $\left(1-F\left(\underline{v}^j\right)^{N-1}\right)B^j$ when she does not submit a bid. The two are equal when $F(\underline{v}^j)^{N-1}\underline{v}^j = c^j$, the result in Eq. (4). Furthermore, under both mechanisms, the threshold depends just on the costs of participation c^j, the number of potential bidders N and the nature of the distribution function $F(v)$.

Table 1 reports the values of this common threshold and the implied non-participation rates as the auction size or number of potential bidders N and the costs of participation c vary for each of the four distributions of private values. One of the first properties of the data to catch our attention was the responsiveness of the threshold to variations in cost. When the distribution of private values is bell-shaped, for example, the difference between $c=0$ or costless participation and $c=0.01$, a cost equal to one fiftieth of the representative potential bidder's private value, is the difference between no threshold and one equal, in the case $N=5$, to 0.46. In other terms, there is now an almost 1 percent ($0.0079=F(0.46)^5=(0.38)^5$) chance that *no one* will want to submit a bid, despite the fact that there are few low value bidders. Whether $c=0.01$ constitutes a small obstacle or not is to some extent a matter of context – if the costs of participation are for the most part psychic, then for a familiar mechanism, costs could well be much lower than this – we were nevertheless struck by how quickly bidders are driven from the auction.[5]

Furthermore, in small auctions, even a small increase in the number of potential bidders induces a substantial increase in the threshold. In the uniform case when $c=0.01$, for example, the threshold rises from 0.10 to 0.40 as N increases from 2 to 5, and when $N=20$, which, for most practical purposes, is still a small auction, the threshold rises to 0.79. To provide a more intuitive characterization of the same phenomenon, *increases in the number of potential bidders*

Table 1
Threshold values and non-participation rates under The FP and AP mechanisms.

		Participation cost = 0.01		Participation cost = 0.05		Participation cost = 0.10	
		Threshold value	Share of inactive bidders	Threshold value	Share of inactive bidders	Threshold value	Share of inactive bidders
(1,1)	N = 2	0.10	0.10	0.22	0.22	0.32	0.32
	N = 5	0.40	0.40	0.55	0.55	0.63	0.63
	N = 10	0.63	0.63	0.74	0.74	0.79	0.79
	N = 20	0.79	0.79	0.86	0.86	0.89	0.89
(2,2)	N = 2	0.17	0.06	0.30	0.17	0.38	0.26
	N = 5	0.46	0.38	0.57	0.54	0.63	0.63
	N = 10	0.63	0.63	0.70	0.75	0.74	0.80
	N = 20	0.74	0.80	0.79	0.86	0.82	0.90
(1,3)	N = 2	0.06	0.17	0.14	0.36	0.20	0.49
	N = 5	0.19	0.48	0.29	0.64	0.35	0.73
	N = 10	0.32	0.68	0.41	0.79	0.46	0.84
	N = 20	0.44	0.82	0.51	0.88	0.56	0.91
(5,1)	N = 2	0.46	0.02	0.61	0.08	0.68	0.15
	N = 5	0.80	0.33	0.87	0.49	0.90	0.58
	N = 10	0.90	0.61	0.94	0.72	0.95	0.78
	N = 20	0.95	0.79	0.97	0.86	0.98	0.89

This table reports the threshold value and share of bidders who are inactive under either the FP or AP mechanism for various numbers of potential bidders and participation costs under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions of private values.

produce small, and ever smaller, increases in the expected number of active bidders, from $3=5(0.6)$ when $N=5$ to $3.7=10(0.37)$ when $N=10$, and then to $4.2=20(0.21)$ when $N=20$. In this particular case, in other words, the addition of 15 more potential bidders caused the expected number of active bidders to increase by little more than one.

There are at least two senses in which the pattern is a robust one. First, while it is possible to construct examples in which, over some interval, the expected number of active bidders *falls* as the number of potential bidders rises, this occurs in none of the cases represented in Table 1.[6] Second, and to our initial surprise, for a fixed participation cost c, the relationship between auction size and the number of active bidders doesn't vary much with the distribution of private values. Consider, for example, the situation in which $c=0.05$ and $N=10$. While the threshold value varies from 0.70 in the auction with few extreme bidders to, on the one hand, 0.41 in the auction with low value bidders or, on the other hand, 0.94 in the auction with high value bidders, the likelihoods of non-participation are, respectively, 0.75, 0.79 and 0.72, consistent with 2.55, 2.08 and 2.78 active bidders. If the auction is then doubled in size, so that $N=20$, the expected numbers of active bidders become 2.71, 2.31 and 2.89.

Table 1 also hints, however, that both the threshold and expected number of active bidders will be sensitive to the costs of participation. When there are 10 potential bidders whose private values are drawn from the uniform distribution, for example, an increase in costs from 0.01 to 0.05 causes the threshold to rise, from 0.63 to 0.74, and the expected number of active bidders to fall, from 3.69 to 2.59. Curiously, perhaps, almost the same number (1.10) of active bidders are "lost" under other distributions: $1.14=3.69-2.55$ when the distribution is bell-shaped, $1.11=3.19-2.08$ when it is skewed to the right, and $1.16=3.94-2.78$ when it is skewed to the left.

A comparison between Eqs. (4) and (5) shows that the participation threshold should be higher, *ceteris paribus*, in second price auctions, and

[4] She pays nothing to acquire the object but, as a result, enjoys no warm glow and, since auction revenues are zero, no common return.

[5] We are grateful to an anonymous reviewer for the reminder that the "size" of these costs cannot be classified *a priori*.

[6] At least one reader has wondered whether this is ever possible. If the addition of one more potential bidder causes an active bidder to withdraw, then wouldn't that bidder have been better off as a non-participant beforehand, too? A simple example, adapted from Menezes and Monteiro (2000), suggests otherwise, however: if F(v |3, 1) = v3 and c = 0.3, for example, there will be 1.21291 active bidders, in expectation, when N = 5, but 1.21262 when N = 4 and 1.21247 when N = 6.

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Table 2
Threshold values and non-participation rates under the SP mechanism.

		Participation cost = 0.01		Participation cost = 0.05		Participation cost = 0.10	
		Threshold value	Share of inactive bidders	Threshold value	Share of inactive bidders	Threshold value	Share of inactive bidders
(1,1)	N = 2	0.00	0.00	0.00	0.00	0.04	0.04
	N = 5	0.21	0.21	0.30	0.30	0.35	0.35
	N = 10	0.44	0.44	0.51	0.51	0.54	0.54
	N = 20	0.62	0.62	0.67	0.67	0.69	0.69
(2,2)	N = 2	0.00	0.00	0.00	0.00	0.01	0.00
	N = 5	0.32	0.20	0.40	0.29	0.44	0.34
	N = 10	0.50	0.43	0.55	0.51	0.57	0.54
	N = 20	0.62	0.62	0.65	0.67	0.66	0.69
(1,3)	N = 2	0.00	0.00	0.03	0.08	0.06	0.18
	N = 5	0.10	0.26	0.14	0.37	0.17	0.42
	N = 10	0.19	0.47	0.23	0.54	0.25	0.57
	N = 20	0.29	0.64	0.32	0.69	0.33	0.70
(5,1)	N = 2	0.00	0.00	0.00	0.00	0.00	0.00
	N = 5	0.70	0.16	0.76	0.25	0.79	0.30
	N = 10	0.84	0.41	0.87	0.49	0.88	0.52
	N = 20	0.91	0.61	0.92	0.66	0.93	0.68

This table reports the threshold value and share of bidders who are inactive under either the SP mechanism (a = 0.25,b = 0.35) for various numbers of potential bidders and participation costs under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions of private values.

this is reflected in Table 2, which reports second price thresholds for various numbers of potential bidders N and costs c. In superficial terms, the difference between the definitions in Eqs. (4) and (5) is the presence of an additional term, $\alpha(N-1)F(\underline{v}^s)^{N-2}\,(1-F(\underline{v}^s))\sigma^s(\underline{v}^s)$, in the latter. In behavioral terms, this is the return to the marginal bidder when there is just one other active bidder, and her (now non-zero) threshold bid determines what the winner pays and, therefore, the common return. This has at least two important implications for empirical work. First and foremost, if costs are the same, the participation rate in second price auctions should exceed that in either first price or all-pay auctions. Second, in second price auctions, the decision to participate is sensitive to the rate of common return α.

The results i`n Table 2 provide some sense of how different the thresholds will be in practice. In the extreme case of $N=2$ potential bidders with low participation costs, there is no threshold at all. That is, both bidders will participate, no matter what their private values. In fact, in auctions with few(er) low value bidders, in particular when the distribution of private values is either $F(v|2,2)$ or $F(v|5,1)$, the threshold is zero even when costs are 0.10. To understand this, recall that in the case $N=2$ – or, with $N>2$ potential bidders, the sub-case in which there are two active bidders – the representative bidder knows that she will either win the auction or determine what the winner pays and therefore the public benefits that accrue to both bidders. This is sometimes sufficient to induce low value bidders to participate, despite the costs.

While full participation is a special feature of (some) "minimal" or $N=2$ second price auctions, the difference remains substantial as auction size increases. In the uniform case, the increase in the threshold under either the first price or all-pay mechanisms, from 0.10 to 0.79, for example, as the number of potential bidders increases from 2 to 20 when costs are 0.01, stands in marked contrast to the increase from 0 to 0.62 under the analogous second price mechanism. In an auction with 20 potential bidders, this is the equivalent of an almost 85% increase in the number of active bidders, from 4.11 to 7.58. The size of this effect is not an artifact of the choice of distribution function: for the same auction size and participation costs, the numbers of expected bidders are 4.06 and 7.56 when the distribution is $F(v|2,2)$, 3.60 and 7.18 when it is $F(v|1,3)$, and 4.27 and 7.74 when it is $F(v|5,1)$. In short, in the absence of cost differentials, it seems that second price auctions will be more "active," and to the extent that this is a secondary objective for the charity, an important point in their favor.

Otherwise, the same broad patterns characterize participation across mechanisms. The expected number of active bidders, for example, is not all that sensitive to the distribution of private values, but is responsive to variations in cost. Under the bell-shaped distribution, for example, the expected number of active bidders when $N=20$ (7.56) and costs are 0.01 is almost identical to that under the uniform (7.58), and not far from those in the right (7.18) and left-skewed (7.74) distributions, but as costs rise to 0.05, the expected number of active bidders falls to 6.66.

3.3. Bid functions

Consider, for comparison purposes, the familiar result that in a first price auction without spillovers or participation costs, bidders whose values are drawn from a uniform distribution will "shade" their bids by an amount equal to $\left(\frac{1}{N}\right)^{\text{th}}$ of their value, and bid $\frac{N-1}{N}v$. This is depicted, for $N=15$, as the solid line in the upper left panel in Fig. 2, in which various first price bid functions have been plotted. Relative to this benchmark, the introduction of revenue proportional benefits ($\alpha=0.25$) and warm glow ($\gamma=0.10$), represented in the same panel by the dotted line, seems to function like an *ad valorem* subsidy to bidders, an observation easily substantiated on the basis of (1): when $\underline{v}=0$ and $F(v)=v$, the bid function is $\frac{N-1}{(1-\gamma)N-\alpha}v$, which is $\frac{\alpha+\gamma N}{(1-\gamma)N-\alpha}$ percent more than would be bid in their combined absence.

Under some conditions, the subsidy is sufficient to reverse bid shading. In the diagram, a bidder whose private value is 1, for example, will bid 1.057; in general, $\sigma^f(v)$ will exceed v under the uniform distribution when $\alpha+\gamma N>1$, an inequality that seems likely to be satisfied in most large auctions. Furthermore, the subsidy is increasing in both the common return α and warm glow γ, as expected, and decreasing in the number of potential bidders N.

The further addition of participation costs equal to 0.05 exerts a dramatic effect on the bid function, as the dashed line in the same panel reveals. The behavior of bidders is now sharply nonlinear, both because bids are undefined below the threshold but also because the bid function is now concave above the threshold. Close to the threshold, bids increase very rapidly and then level off. As a result, the effect of participation costs on the value of the average bid, as opposed to the number of bidders, is quite limited: a bidder who decides to participate knows that if others follow suit, their values must (also) be quite high, and therefore bids aggressively. A bidder whose value is close to the maximum (1), for example, bids almost as much as she would in the absence of participation costs.

The fourth and final function plotted as a series of dots and dashes in the same panel is the equilibrium bid function when the common return, warm glow and participation cost remain in place, but the number of potential bidders is reduced to $N=5$. It underscores the fact that one standard result on auction size and first price bids – that bidders with more competitors are more aggressive because they cannot afford to shade their bids as much – doesn't hold in this environment, at least not for all values. In visual terms, the reason is that the smaller auction also has a lower threshold, so that a bidder who is indifferent about participation when $N=15$, and who would therefore submit a zero bid if she did participate, would find it in her interest to submit a positive bid when $N=5$. For high value bidders, the "shading effect" appears to dominate; for low(er), but still above the second threshold, value bidders, the "participation effect" does, another important consideration in the estimation of bid functions.

The other panels in Fig. 2 show the same four bid functions for the three alternative value distributions, and suggest that these results are robust. Consider what is perhaps the least similar case, the situation depicted in the lower left panel in which there is a

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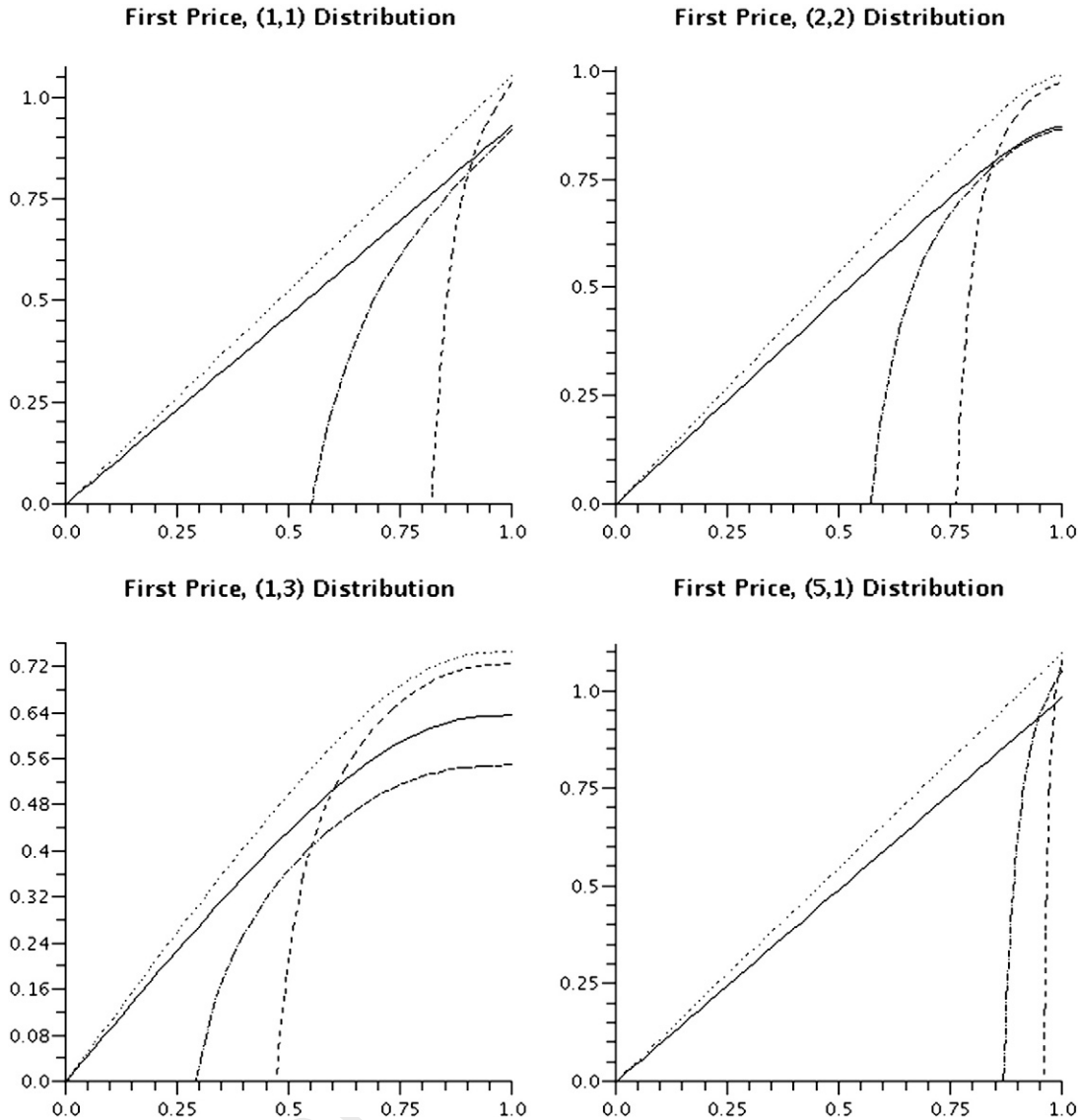

Fig. 2. Optimal bids in FP auctions as a function of private value under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions.

preponderance of low value bidders. It should come as no surprise that even in the standard case – that is, no common return, no warm glow, and no costs of participation – bids are no longer proportional to values: because (small) variations in private value do not have much effect on the likelihood that a high value bidder will win in this environment, bids are not adjusted much either. Furthermore, unlike the uniform case, bidders never bid more than their values, at least for the parameter values considered here.

This said, the two panels share at least three important features. First, it still appears that in the absence of participation costs, the introduction of a common return and warm glow have much the same effect on bids as an *ad valorem* subsidy. Second, those with values close to the maximum aren't much affected by participation costs or, in broader terms, the effects of these costs on bid behavior diminish with value. Third, with both shading and participation effects at work, high and low value bidders respond quite differently to an increase in auction size.

The characterization of second price bid functions is much less complicated. First and foremost, the four panels in Fig. 3 provide visual confirmation that with the common return and warm glow present, variations in the number of potential bidders N or participation costs c influence the participation decision but not, conditional on participation, the bid itself. In effect, there exists a "one size fits all" second price bid function that is "activated" for some combinations of N and c but not others. In the uniform case depicted in the upper left panel, for example, a bidder with private value $v=0.30$ will bid 0.502 when $\alpha=0.25$ and $\gamma=0.10$ when costs c are zero, but not bid (as opposed to a bid of zero) when costs are 0.05, but another bidder with a value just 0.01 higher will bid 0.511 in both situations.

Furthermore, consistent with intuition, this one size fits all bid function differs across distributions but in all cases reflects some inflation of bids relative to the standard auction, in which it is dominant to bid one's value, no matter what the distribution of values. This inflation no longer resembles an *ad valorem* subsidy, however, as it did in first price auctions. Under a uniform distribution, for example, the difference declines not just in proportional, but absolute, terms as value increases, from 0.242 $(=0.242-0.00)$ when $v=0$ to $0.11(=1.11-1.00)$ when $v=1$. The same is true when the distribution of values is either hump shaped or skewed to the left, but not when it is skewed right, when the difference increases from 0.094 when $v=0$ to 0.111 when $v=1$. Since the difference between standard and charity-inflated second price bids does not vary much across distributions for high value bidders – indeed, is the same for bidders with $v=1$ – the explanation is found in the differences for low value bidders.

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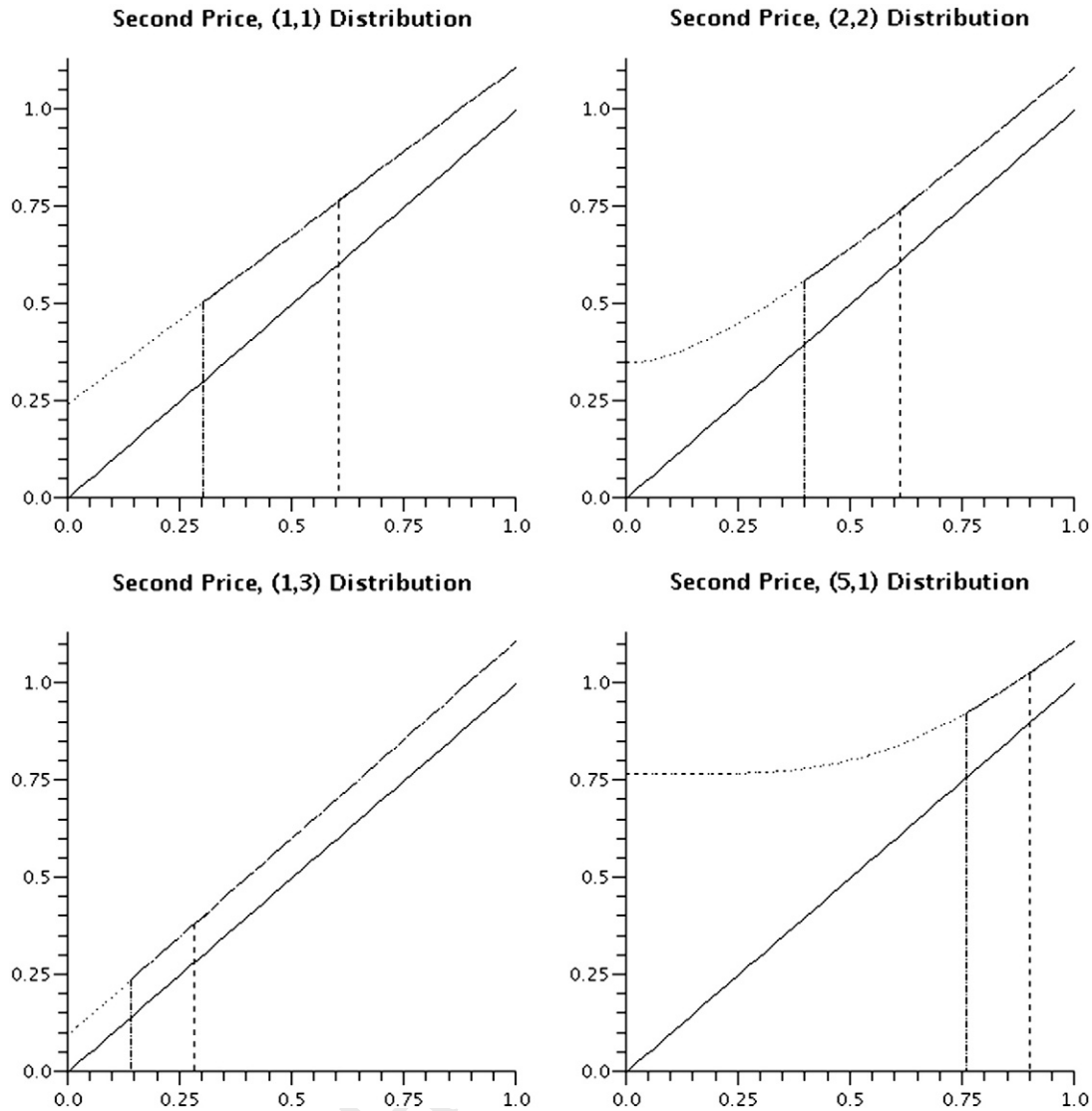

Fig. 3. Optimal bids in SP auctions as a function of private value under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions.

Consider, for example, second price auctions with a preponderance of high value bidders which, as illustrated in the lower right panel of Fig. 3, produces the largest difference in the behavior of low value bidders: a bidder whose value is close to zero will bid almost nothing, for example, in the absence of common return and warm glow, but more than 0.75 in their presence. The intuition is that in the (expected) presence of many high value bidders, the benefits to low value bidders of an inflated bid – in particular, the possible increase in the "second price" and therefore auction revenues and common return – exceed the costs of an improbable "win."

The effects of participation costs on all-pay bids is illustrated in Fig. 4. Consider, for example, the behavior of the median bidder in the case where the distribution of private values is uniform. Since the thresholds under the first price and all-pay mechanisms are the same, we know, for example, that this bidder will not participate when there are $N=15$ potential bidders and costs are equal to 0.05, or one tenth of her private value. It is important to note, however, that even if participation was costless, the optimal bid would be less than one hundredth of one percent of this value or, to be more precise, 4.38×10^{-5}, a bid that is itself a substantial (in proportional terms, at least) increase over the optimal bid in the equivalent non-charity auction, which is 2.85×10^{-5}.

The uniform case also exhibits the predictable bid inflation associated with charity auctions, one that, in this case, increases in absolute, but decreases in relative, terms. It also demonstrates that the common view that increased competition restrains bidders when bids are forfeited does not hold in the presence of participation costs.[7] In this case, the upper left panel of Fig. 4 reveals that high value bidders, at least, are more aggressive when $N=15$ than $N=5$. In broader terms, the difference in thresholds causes the bid functions to cross once, a pattern reminiscent of first price auctions: for low(er) values in their common domain, bids are smaller with $N=15$ than $N=5$, while the opposite is true for high(er) values.

Unlike the first price auction, however, even the behavior of very high value bidders is sensitive to the existence of participation costs. The so-called "maximal bidder" will bid 1.36 in a charity auction with participation costs of 0.05, and 1.44 in the same auction without such costs.

[7] In fact, it doesn't hold in their absence, either: from Eq. (3), the optimal bid function when c, and therefore $\underline{v}$, are zero, is $\frac{N-1}{N}\frac{1}{1-\beta}v^N$, the value of which must only eventually decline in N.

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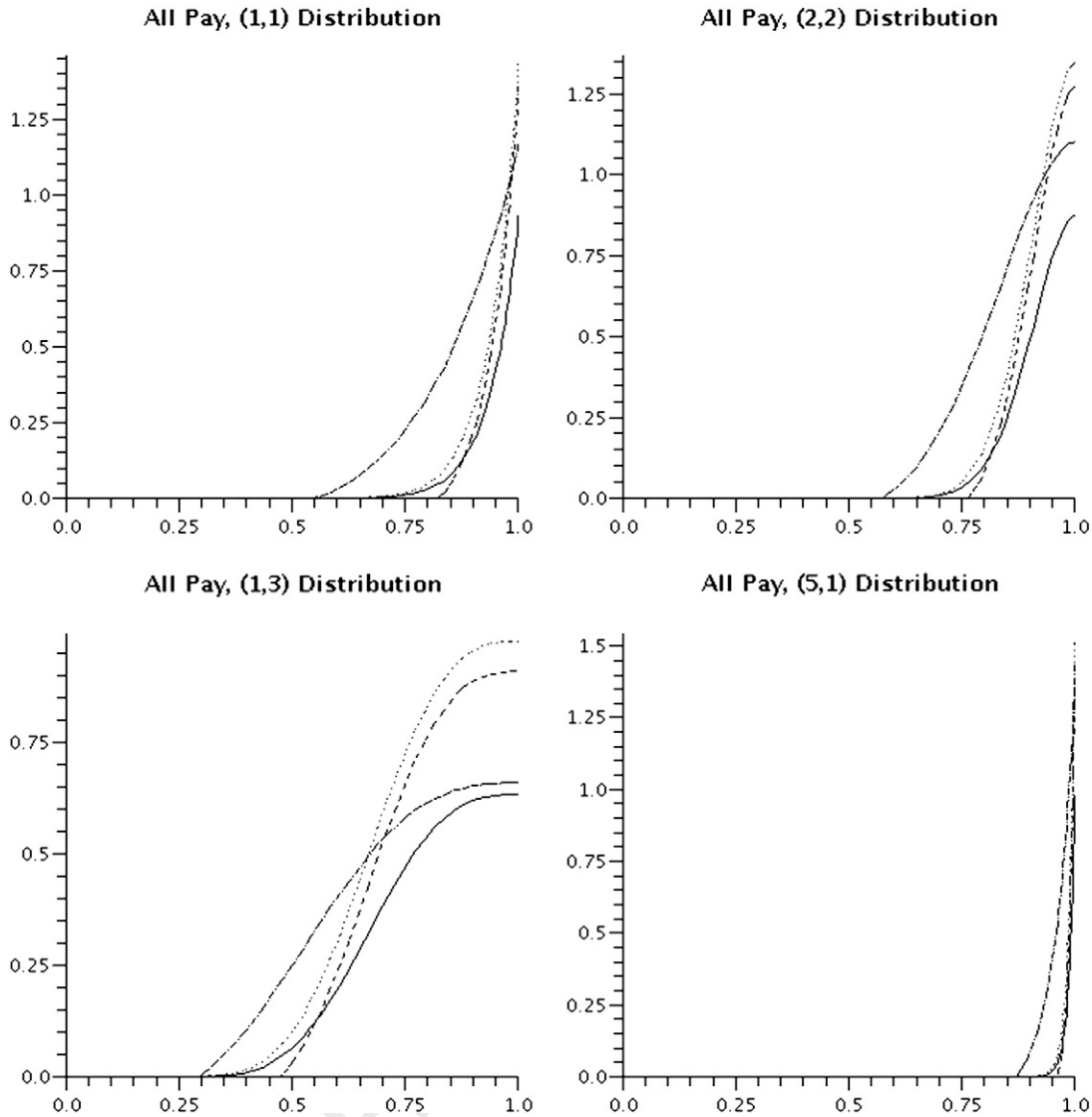

Fig. 4. Optimal bids in AP auctions as a function of private value under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions.

All of these features are robust with respect to the distribution of private values, or at least the four distributions considered here.

Finally, Fig. 5 allows for the comparison of bid functions across mechanisms and distributions in the special, if now familiar, case of $N=15$ potential bidders, participation costs $c=0.05$, common return $\alpha=0.25$ and warm glow $\gamma=0.10$. The surprise, perhaps, is how little can be said about the relative sizes of bids across mechanisms. One obvious exception is that for all values in their common domain, second price bidders bid strictly more than their first price counterparts, a result that carries over from standard auctions. It is not even the case that both are always more aggressive than those who must forfeit their bids under the all-pay format; in fact, for three of the four distributions pictured here, those with very high values will bid more in all-pay than *either* first or second price auctions. The intuition for this is that with revenue proportional benefits, such bidders are, in effect, subsidized by their rivals. This is consistent with the observation that the exception is the distribution associated with a preponderance of low value bidders, depicted in the lower left panel: under these conditions, the common return is never sufficient to rationalize bids well in excess of private values.

This said, under all four distributions, all-pay bids are smallest for low(er) value bidders, and remain so over much of the common domain before surpassing (at least) first price bids, a consequence of the fact that all-pay bidders forfeit their bids, no matter what the outcome of the auction.

3.4. Revenue functions

Our principal interests here are not the bid function themselves, but their revenue implications. To this end, consider Fig. 6, which plots the variation in expected revenue as a function of auction size (N) across both distributions and mechanisms. Its most obvious feature is that in every case, revenue rises, at a diminishing rate, with the number of potential bidders.[8]

Furthermore, with the limited exception of the $F(v|1,3)$ distribution, expected revenue more or less levels off after the first dozen or so potential bidders. A similar pattern characterizes the standard auction, but the explanation is a little different. In the standard case, the first order statistic for private values is a concave function of the number of bidders with an upper limit of 1, the upper bound of the distribution of values, but in charity

[8] It remains to be seen, therefore, whether the example in Menezes and Monteiro (2000) of a revenue function that, after some point, declines in N, is a practical one.

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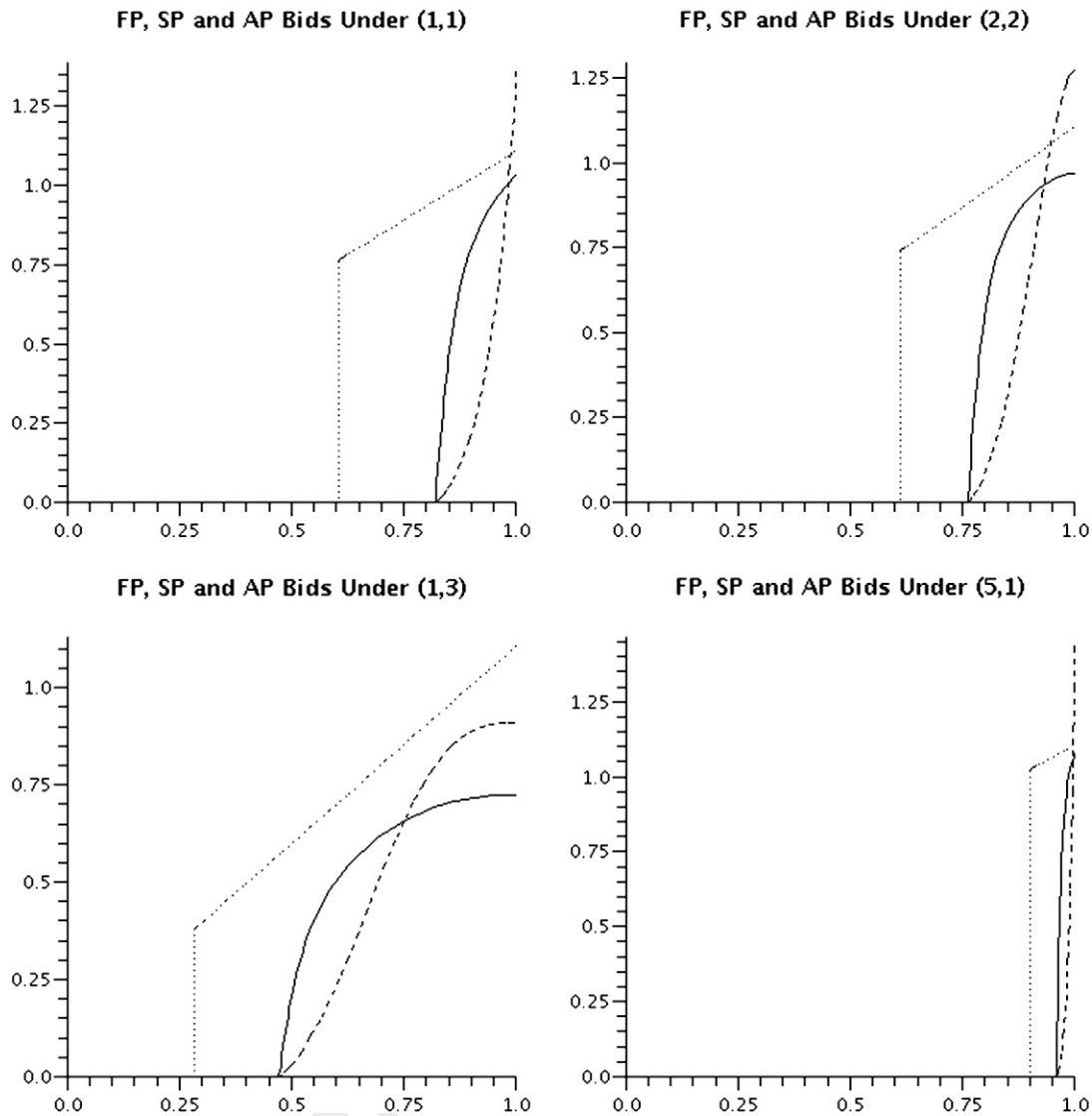

Fig. 5. Optimal bids in FP, SP and AP auctions under uniform (1,1), hump-shaped (2,2), right-skewed (1,3) and left-skewed (5,1) distributions.

auctions with endogenous participation, this is amplified by the fact that as auction size increases, the number of active bidders also increases at an ever diminishing rate. The map from potential to active bidders also helps to explain the fact that revenues in the low value $F(v|1,3)$ auction do not level off as soon: as a review of Tables 1 and 2 reveals, there are fewer active bidders, *ceteris paribus*, in this environment.

Some will be surprised that even with $N=40$ potential bidders, the first and second price mechanisms produce such different revenue. The problem is that here, too, intuition is based on the case of compact distributions and costless participation. From Eqs. (1) and (2), it follows that in both cases, the winner's payment, and therefore auction revenue, are equal to $\sigma^f(1)=\sigma^s(1)=(1-\gamma)^{-1}$, no matter what the distribution of values.

This leads us to broader conclusions about the relative performance of mechanisms. Fig. 6 suggests that at least two inequalities are robust with respect to the distribution of private values. For any number of potential bidders N, both the second price and all pay formats "revenue dominate" their first price equivalent. Both inequalities are consistent with previous results for auctions with a fixed number of active bidders (that is, costless participation) and have the same intuition.

The response of the second price/all pay revenue differential to variations in the number of potential bidders is more complicated, but not much so. Under all four distributions, the all pay mechanism *eventually* produces more revenue, in expectation, than its second price equivalent. For auctions with either a uniform or bell-shaped distribution of values, it happens almost at once – that is, when there are 3 or more potential bidders – and for the auction with a preponderance of high value bidders, it holds even in the limiting case $N=2$. It is only when there is a preponderance of low value bidders that the second price mechanism does better in auctions of intermediate size (under the assumed parameter values, N less than 30). To understand this, recall that with so many low value bidders, high value bidders aren't subsidized enough to bid very aggressively.

Fig. 7, which depicts the relationship(s) between expected revenue and participation costs for auctions with $N=10$ potential bidders, leads to some important, if unexpected, conclusions. Consistent with intuition, revenues decline as participation costs rise, across both distributions and auction formats. In the case of second price auctions, however, the decline is almost imperceptible: if private values are uniformly distributed, for example, expected revenue declines from 0.953 when $c=0$ to 0.937 when

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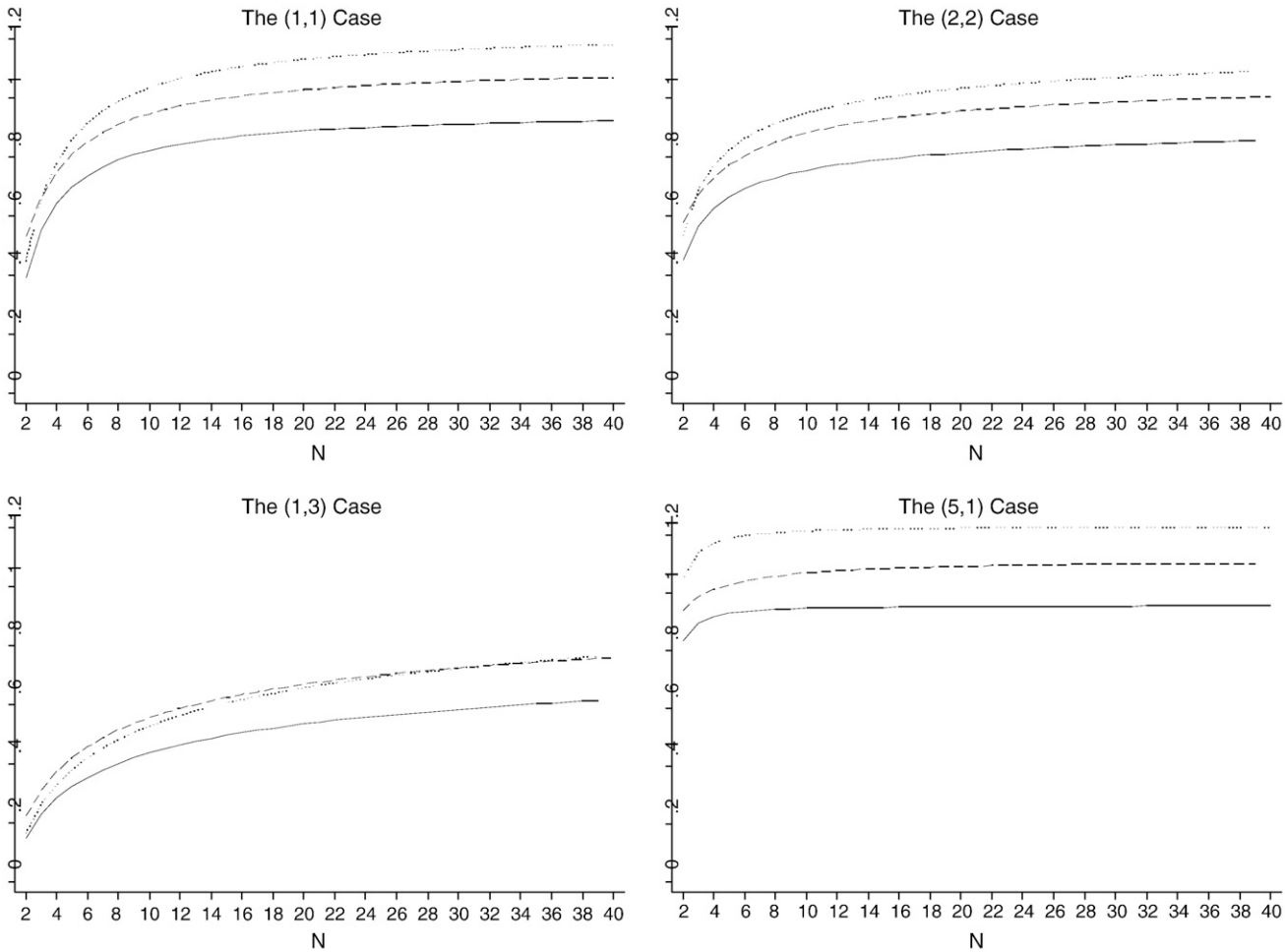

Fig. 6. Expected revenue as a function of the number of potential bidders, with $\alpha = 0.25$, $\beta = 0.35$ and $c = 0.05$. Legend: FP — solid line, SP — dashed line, AP — dotted line.

$c = 0.15$, or 30 percent of the median value. From an operational perspective, charities that do not know what it costs to participate in their auctions will sometimes find that the second price mechanism serves them best, despite the results in Fig. 6. To understand this, recall that in second price auctions, cost influences the decision to participate but not, conditional on participation, the bid itself.

The fact that the all pay mechanism is (much) more cost sensitive than the second price leads to an important reversal: consistent with intuition, the all pay format is more lucrative for charities when there are no, or even few, obstacles to participation, but as participation becomes more difficult, the premium shrinks and is eventually reversed. Both, however, do better than the first price mechanism no matter what the costs of participation.

4. Relationship to previous empirical work

Our immediate purpose here is to provide a theoretical framework for the analysis of endogenous participation in charity auctions, but it is helpful to consider the possible implications of the model for previous empirical work. It should be emphasized, however, that the exercise is a speculative one: it assumes, for example, that bidders submit their equilibrium bids, a matter of considerable debate itself. This said, the lab experiments of Davis et al. (2006) and Schram and Onderstal (forthcoming), for example, which find that raffles and all-pay auctions do well, are consistent with the interpretation of "fixed N designs" as environments with zero participation cost. Our model also predicts that notwithstanding the dramatic effects of even small costs on participation thresholds and therefore individual bid functions, this result should be robust with respect to the introduction of a small common cost.

Under the same assumptions, the model also tells us that on its own, endogenous participation cannot explain the underperformance of the all-pay mechanism in Carpenter et al. (2008) field experiment. They found that there were more active bidders under the first price format than either the second price or all-pay which implies that the order participation thresholds satisfies $\underline{v}^f < \underline{v}^s < \underline{v}^a$. In the absence of cost differentials across mechanisms, however, the model implies, and Fig. 7 illustrates, that more bidders will participate in second price auctions than either first price or all-pay auctions, that is, $\underline{v}^s < \underline{v}^f = \underline{v}^a$. To be consistent with the equilibrium predictions of our model requires, at a minimum, that participation costs in first price auctions be smaller than either alternative.

The further observation that the second price and all-pay mechanisms in Carpenter et al. (2008) produced about the same revenue, and that both produced less than the first price, implies that participation costs in first price auctions c^f are smaller than in the other mechanisms. The implications of their revenue data for c^s and c^a are harder to pin down, but Fig. 7 also hints that unless the costs of participation are implausibly large, the two mechanisms would not produce the same equilibrium revenue under a diverse set of conditions unless it cost bidders more to participate in the all-pay. In short, then, a reconciliation of the field data with the model requires that $c^a > c^s > c^f$.

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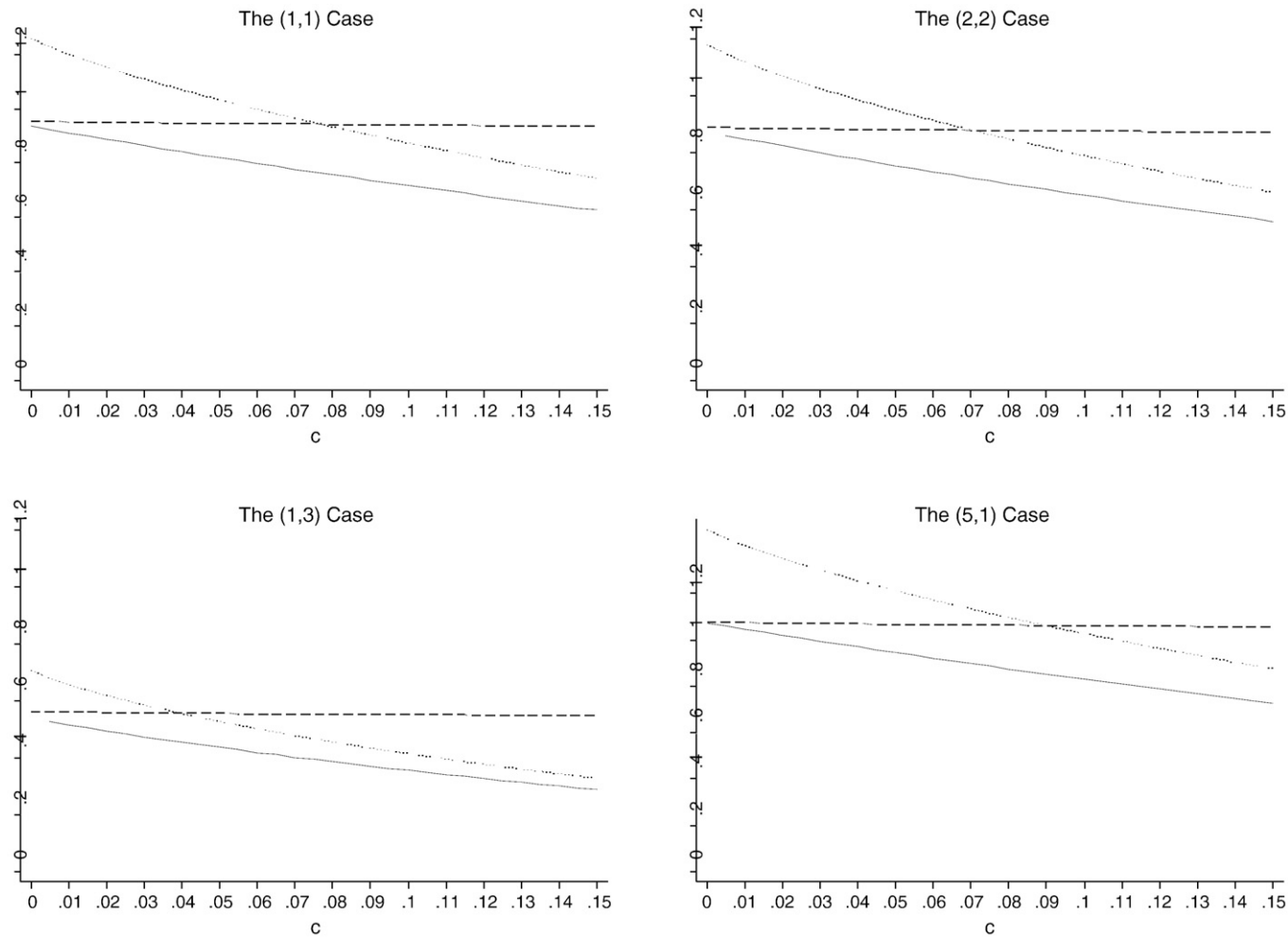

Fig. 7. Expected revenue as a function of participation cost, with $\alpha = 0.25$, $\beta = 0.35$ and $N = 10$. Legend: FP — solid line, SP — dashed line, AP — dotted line.

5. Conclusion

The framework described here calls to mind a number of opportunities, both theoretical and empirical, for further research. The recent lab experiments outlined in Carpenter et al. (2010), for example, offer a first look at the effects of controlled variation in participation costs. It is clear, however, that there remains much work to do on mechanism-specific differences in costs. For example, are there substantial differences in the costs of bid preparation or cognitive costs? Are some mechanisms perceived to be fairer than others?

The model itself does not allow for variation in participation costs across bidders, one of several possible asymmetries that merit attention. Some preliminary work by Bos (2008), for example, suggests that if the distributions from which bidders' private values are drawn are sufficiently different, all-pay auctions will not do well. In a similar vein, while the bidders in our model are risk neutral, it seems reasonable to expect that risk aversion, and differences in risk aversion across bidders, will affect the relative performance of charity auction mechanisms. Last but not least, we do not know much about the effects of behavioral biases and "bidder heuristics" on charity auctions.

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Appendix A. Bid and revenue functions for "Endogenous participation in charity auctions"

This section derives the equilibrium bid and revenue functions for the first price sealed bid, second price sealed bid and all-pay charity auctions, the basis for Propositions 1 and 2 in the paper.

1. First price sealed bid

The representative bidder must decide whether or not to participate and, if she does, what type $\hat{v}$ to announce or, equivalently, what bid $\sigma^f(\hat{v})$ to submit. To this end, consider first the conditions under which someone with the private value $v \geq \underline{v}$ will find it optimal to reveal her true type when the participation threshold v is assumed fixed. With likelihood $C_M^{N-1} F(\underline{v})^{(N-1)-M}(1-F(\underline{v}))^M$, where $C_q^p = \frac{p!}{(p-q)!q!}$, she will compete with M other bidders for the object and, conditional on $M \geq 1$, the first order statistic of their values (that is, the maximum) has the distribution function $G(x, M) = (F(x) - F(\underline{v}))^M / (1-F(\underline{v}))^M$ If $M = 0$, there will of course be no rivals and, therefore, no first order statistic. The conditional return on the bid $\sigma^f(\hat{v})$ for fixed $M \geq 1$ is then:

$$EU(\hat{v}, v, M) = \int_{\underline{v}}^{\hat{v}} \left(v - (1-\beta)\sigma^f(\hat{v})\right) g(x, M)dx + \alpha \int_{\hat{v}}^{\overline{v}} \sigma^f(x) g(x, M)dx \quad (1)$$

where $g(x, M) = dG(x, M) / dx = M(F(x) - F(\underline{v}))^{M-1} f(x) / (1-F(\underline{v}))^M$

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is the conditional density function of the first order statistic. The first term in (1) represents the bidder's expected return when she wins the auction – because she earns both the common return $\alpha\sigma^f(\hat{v})$ on her bid and experiences the warm glow $\gamma\sigma^f(\hat{v})$ in this case, her "net bid" is $(1-(\alpha+\gamma))\sigma^f(\hat{v}) = (1-\beta)\sigma^f(\hat{v})$ – while the second term is the expected benefit that still accrues to her when she loses.

It follows that the unconditional expected return, $EU(\hat{v}, v)$, will be:

$$\begin{aligned} EU(\hat{v},v) &= F(\underline{v})^{N-1}(v-(1-\beta)\sigma^f(\hat{v})) \\ &\quad + \textstyle\sum_{M=1}^{N-1} C_M^{N-1} F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M EU(\hat{v},v,M) \\ &= F(\underline{v})^{N-1}(v-(1-\beta)\sigma^f(\hat{v})) \\ &\quad + (v-(1-\beta)\sigma^f(\hat{v}))\textstyle\sum_{M=1}^{N-1} C_M^{N-1} F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M \int_{\underline{v}}^{\hat{v}} g(x,M)dx \\ &\quad + \alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1} F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M \int_{\hat{v}}^{\bar{v}} B^F(x)g(x,M)dx \\ &= F(\underline{v})^{N-1}(v-(1-\beta)\sigma^f(\hat{v})) + (F(\hat{v})^{N-1}-F(\underline{v})^{N-1})(v-(1-\beta)\sigma^f(\hat{v})) \\ &\quad + \alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1} F(\underline{v})^{N-1-M} M\int_{\hat{v}}^{1}(F(x)-F(\underline{v}))^{M-1} f(x)\sigma^f(x)dx \end{aligned} \tag{2}$$

after substitution for $G(x,M)$ and $g(x,M)$, where the first term on the right hand side of each equality is the expected return in the case where there are no other bidders, and the last equality follows from the fact that $\int_{\underline{v}}^{\hat{v}} g(x,M) = (F(\hat{v})-F(\underline{v}))^M / (1-F(\underline{v}))^M$ and that, as a consequence of the binomial theorem, $\sum_{M=1}^{N-1} C_M^{N-1} F(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^M = F(\hat{v})^{N-1}-F(\underline{v})^{N-1}$.

The derivative of $EU(\hat{v}, v)$ with respect to the bidder's choice variable $\hat{v}$ is therefore:

$$\begin{aligned} \frac{\partial EU(\hat{v},v)}{\partial \hat{v}} &= -(1-\beta)F(\hat{v})^{N-1}\frac{d\sigma^f(\hat{v})}{d\hat{v}} \\ &\quad + (N-1)F(\hat{v})^{N-2}f(\hat{v})(v-(1-\beta)\sigma^f(\hat{v})) \\ &\quad -\alpha f(\hat{v})\sigma^f(\hat{v})\textstyle\sum_{M=1}^{N-1} C_M^{N-1} MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} \\ &= -(1-\beta)F(\hat{v})^{N-1}\frac{d\sigma^f(\hat{v})}{d\hat{v}} + (N-1)F(\hat{v})^{N-2}f(\hat{v})(v-(1-\beta)\sigma^f(\hat{v})) \\ &\quad -\alpha(N-1)F(\hat{v})^{N-2}f(\hat{v})\sigma^f(\hat{v}) \end{aligned} \tag{3}$$

where the second line follows from a corollary of the binomial theorem, $\sum_{M=1}^{N-1} C_M^{N-1} MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} = (N-1)F(\underline{v})^{N-2}$. The first order condition for a SBNE is that $\frac{\partial EU(\hat{v},v)}{\partial \hat{v}} = 0$ at $\hat{v} = v$, which leads, after some simplification, to the first order differential equation:

$$\frac{d\sigma^f(v)}{dv} + \frac{(N-1)(1-\gamma)}{(1-\beta)}\frac{f(v)}{F(v)}\sigma^f(v) = \frac{(N-1)}{(1-\beta)}\frac{f(v)}{F(v)}v \tag{4}$$

While Eq. (4) is not exact, there exists an integrating factor, $F(v)^\theta$, where $\theta = \frac{(N-1)(1-\gamma)}{(1-\beta)}$, so that:

$$\frac{d(\sigma^f(v)F(v)^\theta)}{dv} = \frac{N-1}{1-\beta}F(v)^{\theta-1}f(v)v \tag{5}$$

or:

$$\sigma^f(v)F(v)^\theta = \frac{N-1}{1-\beta}\int F(x)^{\theta-1}f(x)xdx + k \tag{6}$$

where k is a constant of integration. Because the optimal threshold bid, $\sigma^f(\underline{v})$, and therefore the product $\sigma^f(\underline{v})F(\underline{v})^\theta$, are both zero, it follows that[9]:

$$\sigma^f(v) = \frac{N-1}{(1-\beta)}\frac{1}{F(v)^\theta}\int_{\underline{v}}^{v} F(x)^{\theta-1}f(x)xdx \tag{7}$$

or, after integration by parts and further simplification:

$$\sigma^f(v) = \frac{1}{1-\gamma}\left[v - \frac{F(\underline{v})^\theta}{F(v)^\theta}\underline{v} - \frac{1}{F(v)^\theta}\int_{\underline{v}}^{v} F(x)^\theta dx\right] \tag{8}$$

Inasmuch as the participation threshold is not predetermined, however, the optimal bid function (8) is not a reduced form. To this end, recall that the revenue proportional benefits of the auction are not conditional on participation, and observe that a potential bidder with private value v should be indifferent between participation (and the submission of a zero bid) and non-participation. If such a bidder does participate, the likelihood that she will win the auction is $F(\underline{v})^{N-1}$, in which case she receives a benefit equal to her private value $\underline{v}$. (Since $\sigma^f(\underline{v}) = 0$, there is neither a common return nor a warm glow.) With likelihood $C_M^{N-1}F(\underline{v})^{(N-1)-M}(1-F(\underline{v}))^M$, on the other hand, she will lose the auction to one of $M \geq 1$ other bidders, but receive a benefit that is equal to a fraction α of the expected maximum bid, or $\alpha\int_{\underline{v}}^{1} g(x,M)\sigma^f(x)dx$. The net benefit of participation is therefore:

$$F(\underline{v})^{N-1}v + \alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{(N-1)-M}(1-F(\underline{v}))^M\int_{\underline{v}}^{1} g(x,M)\sigma^f(x)dx - c^f \tag{9}$$

where c^f is the cost of participation in a (f)irst price auction. The net benefit of non-participation is equal to:

$$\alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{(N-1)-M}(1-F(\underline{v}))^M\int_{\underline{v}}^{1} g(x,M)\sigma^f(x)dx \tag{10}$$

since the externalities that other bidders produce are not limited to participants. The "threshold bidder" is therefore someone for whom:

$$F(\underline{v})^{N-1}\underline{v} = c^f \tag{11}$$

This condition defines an implicit function in which the participation threshold v depends on the costs of participation c^f, the number of potential bidders N and, implicitly, the shape of the distribution function $F(v)$. If the effects of the first are more or less predictable – if potential bidders have better outside options, fewer of them will participate – the implications of the second are more subtle and call for some comment. As the number of potential bidders increases, so, too, does the likelihood that a particular active bidder will lose whatever she has "invested" in the auction which, in turn, causes the threshold to rise. It is then not obvious that an increase in the number of potential bidders or, if one prefers, auction size, will always lead to an increase in the expected number of active bidders and, so, expected revenue.

It is important to note, however, that this participation effect is *not* the result of some increased desire to free ride on the contributions of other bidders. The threshold $\underline{v}$ in (11) does not depend on either the common return α or warm glow γ: it is the same condition, in fact, that Menezes and Monteiro (2002) derive for their "no spillover" model. The reason is that non-participants benefit from these spillovers, too.

Charities will be less interested in bid functions and their properties than expected revenue R^f and, to this end, we note that since the density function of the first order statistic for all N private

[9] In Engers and McManus (2006), the optimal bid at the "threshold" – in their case, the lower limit on the compact support of F – is indeterminate. The difference is that, in their case, the likelihood that a bidder with the threshold value wins the auction is zero.

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values is $NF(v)^{N-1}f(v)$, R^f will be equal to:

$$R^f = N\int_{\underline{v}^f(c^f,N)}^1 F(v)^{N-1}f(v)\sigma^f(v)\,dv \tag{12}$$

where the threshold value is written $\underline{v}^f(c^f,N)$ as a reminder that the lower limit is not fixed in the usual sense.

2. Second price sealed bid auction

The derivation of the SBNE bid and expected revenue functions in the second price auction calls for the introduction of another distribution function, $J(x,M)$, the conditional distribution of the second order statistic for private values when there are $M\geq 2$ other active bidders:

$$J(x,M) = M\left(\frac{F(x)-F(\underline{v})}{1-F(\underline{v})}\right)^{M-1} - (M-1)\left(\frac{F(x)-F(\underline{v})}{1-F(\underline{v})}\right)^{M} \tag{13}$$

It will also be useful to note that the likelihood that a bidder who announces type $\hat{v}$ is the runner-up is:

$$M\left(\frac{F(\hat{v})-F(\underline{v})}{1-F(\underline{v})}\right)^{M-1}\left(1-\frac{F(\hat{v})-F(\underline{v})}{1-F(\underline{v})}\right) = \frac{M(F(\hat{v})-F(\underline{v}))^{M-1}(1-F(\hat{v}))}{(1-F(\underline{v}))^{M}} \tag{14}$$

since it is her bid, $\sigma^s(\hat{v})$, that determines the winner's payment.

With this in mind, with likelihood $F(\underline{v})^{N-1}$, where $\underline{v}$ once more denotes the relevant participation threshold, the representative bidder will have no active competitors. If it is assumed that in an auction with one bidder, the "second price" is zero, then such a bidder would earn a benefit of v, no matter what bid $\sigma^s(\hat{v})$ she submits.

With likelihood $(N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))$, on the other hand, she will compete with just one other bidder ($M=1$), with expected benefits equal to:

$$\begin{aligned} EU(\hat{v},v,1) &= \int_{\underline{v}}^{\hat{v}} (v-(1-\beta)\sigma^s(x))g(x,1)dx + \frac{(1-F(\hat{v}))}{(1-F(\underline{v}))}\alpha\sigma^s(\hat{v}) \\ &= \frac{1}{(1-F(\underline{v}))}\int_{\underline{v}}^{\hat{v}} (v-(1-\beta)\sigma^s(x))f(x)dx + \frac{(1-F(\hat{v}))}{(1-F(\underline{v}))}\alpha\sigma^s(\hat{v}) \end{aligned} \tag{15}$$

The first term is the (conditional on $M=1$) expected benefit when she wins – the difference between this term and its equivalent under the first price mechanism is that the relevant bid is now $\sigma^s(x)$ rather than $\sigma^s(\hat{v})$ – and the second captures the fact that when she loses, the value of her bid, $\sigma^s(\hat{v})$, determines the winner's payment and therefore the value of the common benefit.

Finally, she will face $M\geq 2$ competitors with likelihood $C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^{M}$, with expected benefits:

$$\begin{aligned} EU(\hat{v},v,M) = &\int_{\underline{v}}^{\hat{v}} (v-(1-\beta)\sigma^s(x))g(x,M)dx \\ &+ \frac{M(F(\hat{v})-F(\underline{v}))^{M-1}(1-F(\hat{v}))}{(1-F(\underline{v}))^{M}}\alpha\sigma^s(\hat{v}) \\ &+ \alpha\int_{\hat{v}}^{1}\sigma^s(x)j(x,M)dx \end{aligned} \tag{16}$$

where:

$$j(x,M) = \frac{dJ(x,M)}{dx} = \frac{M(M-1)(F(x)-F(\underline{v}))^{M-2}(1-F(x))f(x)}{(1-F(\underline{v}))^{M}} \tag{17}$$

is the density function of the second order statistic. As before, the first and second terms represent, respectively, the expected benefits when she wins, and when she loses but submits the second highest bid. The additional third term measures the direct spillover when she is neither the first nor second price bidder.

With some simplification, the unconditional return $EU(v,\hat{v})$ can then be written:

$$\begin{aligned} EU(v,\hat{v}) = &F(\underline{v})^{N-1}v + (N-1)F(\underline{v})^{N-2}\int_{\underline{v}}^{\hat{v}}(v-(1-\beta)\sigma^s(x))f(x)dx \\ &+ \alpha(N-1)F(\underline{v})^{N-2}(1-F(\hat{v}))\sigma^s(\hat{v}) \\ &+ \textstyle\sum_{M=2}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}M\left(\int_{\underline{v}}^{\hat{v}}(v-(1-\beta)\sigma^s(x))(F(x)-F(\underline{v})^{M-1}f(x)dx\right) \\ &+ \alpha(1-F(\hat{v}))\sigma^s(\hat{v})\textstyle\sum_{M=2}^{N-1} C_M^{N-1}MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} \\ &+ \alpha\textstyle\sum_{M=2}^{N-1} C_M^{N-1}M(M-1)F(\underline{v})^{N-1-M} \\ &\times\left(\int_{\hat{v}}^{1}\sigma^s(x)(F(x)-F(\underline{v}))^{M-2}(1-F(x))f(x)dx\right) \end{aligned} \tag{18}$$

The effects of variation in $\hat{v}$ on $EU(v,\hat{v})$ are a little easier to calculate than first seem because the derivatives of the fifth and sixth terms each contain, with opposite signs, the term $\alpha(1-F(\hat{v}))\sigma^s(\hat{v})\sum_{M=2}^{N-1} C_M^{N-1}M(M-1)F(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-2}f(\hat{v})$. It follows that:

$$\begin{aligned} \frac{\partial EU(v,\hat{v})}{\partial\hat{v}} = &(N-1)F(\underline{v})^{N-2}(v-(1-\beta)\sigma^s(\hat{v}))f(\hat{v}) \\ &+ \alpha(N-1)F(\underline{v})^{N-2}[(1-F(\hat{v}))\frac{d\sigma^s(\hat{v})}{d\hat{v}} - \sigma^s(\hat{v})f(\hat{v})] \\ &+ (v-(1-\beta)\sigma^s(\hat{v}))f(\hat{v})\textstyle\sum_{M=2}^{N-1}MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} \\ &+ \alpha[(1-F(\hat{v}))\frac{d\sigma^s(\hat{v})}{d\hat{v}} - f(\hat{v})\sigma^s(\hat{v})]\textstyle\sum_{M=2}^{N-1}MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} \end{aligned} \tag{19}$$

The observation that, as a further consequence of the binomial theorem, $\sum_{M=2}^{N-1}MF(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^{M-1} = (N-1)(F(\hat{v})^{N-2} - F(\underline{v})^{N-2})$, and the requirement that $\partial EU(v,\hat{v})/\partial\hat{v} = 0$ at $v=\hat{v}$ in equilibrium allows the first order condition to be rewritten as:

$$\begin{aligned} 0 = &(N-1)F(\underline{v})^{N-2}(v-(1-\beta)\sigma^s(v))f(v) \\ &+ \alpha(N-1)F(\underline{v})^{N-2}[(1-F(v))\frac{d\sigma^s(v)}{dv} - \sigma^s(v)f(v)] \\ &+ (N-1)(v-(1-\beta)\sigma^s(v))f(v)(F(v)^{N-2} - F(\underline{v})^{N-2}) \\ &+ \alpha(N-1)[(1-F(v))\frac{d\sigma^s(v)}{dv} - f(v)\sigma^s(v)](F(v)^{N-2} - F(v)^{N-2}) \end{aligned}$$

or, after dividing both sides by $(N-1)$ and collecting terms:

$$\begin{aligned} 0 = &(v-(1-\beta)\sigma^s(v))F(v)^{N-2}f(v) \\ &+ \alpha(1-F(v))F(v)^{N-2}\frac{d\sigma^s(v)}{dv} - \alpha\sigma^s(v)F(v)^{N-2}f(v) \end{aligned}$$

which, if $v\neq 0$, so that $F(v)^{N-2}\neq 0$, produces:

$$(v-(1-\beta)\sigma^s(v))f(v) + \alpha[(1-F(v))\frac{d\sigma^s(v)}{dv} - f(v)\sigma^s(v)] = 0 \tag{20}$$

or, if $v\neq 1$ and $\alpha\neq 0$, the first order differential equation[10]:

$$\frac{d\sigma^s(v)}{dv} - \frac{(1-\gamma)}{\alpha}\frac{f(v)}{(1-F(v))}\sigma^s(v) = -\frac{1}{\alpha}\frac{f(v)}{(1-F(v))}v \tag{21}$$

[10] If there is no common return – that is, if $\alpha=0$ – then Eq. (20) collapses to $\sigma^s(v) = (1-\gamma)^{-1}v$, a variation on the standard proposition that in a second price auction with independent private values, individuals will bid these values. In this case, individuals bid $\gamma(1-\gamma)^{-1}$ percent more than their values because it is possible, at least in principle, that there remains a warm glow γ.

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Multiplication of both sides of Eq. (21) by the integrating factor $(1-F(v))^{\frac{1-\gamma}{\alpha}}$ then produces:

$$\frac{d((1-F(v))^{\frac{1-\gamma}{\alpha}}\sigma^s(v))}{dv} = -\frac{1}{\alpha}vf(v)(1-F(v))^{\frac{1-\beta}{\alpha}} \tag{22}$$

or:

$$(1-F(v))^{\frac{1-\gamma}{\alpha}}\sigma^s(v) = -\frac{1}{\alpha}\int vf(v)(1-F(v))^{\frac{1-\beta}{\alpha}} + k \tag{23}$$

where k is the constant of integration.

The choice of boundary condition, and therefore the calculation of k, is complicated for two reasons. The optimal threshold bid $\sigma^s(\underline{v})$ is, for reasons noted earlier, indeterminate, but the derivation of Eq. (23) assumed that $v \neq 1$. The second problem can be circumvented if the domain of $(1-F(v))^{\frac{1-\gamma}{\alpha}}\sigma^s(v)$ is (re)extended such that $(1-F(1))^{\frac{1-\gamma}{\alpha}}\sigma^s(1)$ assumes its limit value of 0. It then follows that:

$$(1-F(v))^{\frac{1-\gamma}{\alpha}}\sigma^s(v) = \frac{1}{\alpha}\int_v^1 xf(x)(1-F(x))^{\frac{1-\beta}{\alpha}}dx. \tag{24}$$

Integration by parts then implies:

$$(1-F(v))^{\frac{1-\gamma}{\alpha}}\sigma^s(v) = \frac{1}{1-\gamma}(1-F(v))^{\frac{1-\gamma}{\alpha}}v + \frac{1}{1-\gamma}\int_v^1 (1-F(x))^{\frac{1-\gamma}{\alpha}}dx \tag{25}$$

or, if one assumes, once more, that $v \neq 1$, so that both sides can be divided by $(1-F(v))^{\frac{1-\gamma}{\alpha}}$:

$$\sigma^s(v) = \frac{1}{1-\gamma}v + \frac{1}{(1-\gamma)(1-F(v))^{\frac{1-\gamma}{\alpha}}}\int_v^1 (1-F(x))^{\frac{1-\gamma}{\alpha}}dx. \tag{26}$$

The limit bids $\sigma^s(\underline{v})$ and $\sigma^s(1)$ are then chosen so that $\sigma^s(v)$ is continuous over the entire interval $[\underline{v}, 1]$.

It isn't difficult to infer from Eq. (26) that, conditional on participation, neither the introduction of spillover effects nor participation costs causes bidders to become "N sensitive." This should not come as much of a surprise, however, because Menezes and Monteiro (2002) show that it is (still) dominant to bid one's value in the absence of the former, while Engers and McManus (2006) determine that in a second price charity auction with a fixed number of bidders, the optimal bid is independent of N.

Menezes and Monteiro (2002) also found, however, that the participation thresholds for first and second price auctions were equal, a result that is *not* robust with respect to the presence of a common return. To understand the difference, consider, once more, the situation faced by the "threshold bidder." If she participates, then with likelihood $F(\underline{v})^{N-1}$ she alone will submit a bid, and therefore win the object worth v to her at a cost of 0, since there is no second price. With likelihood $(N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))$, on the other hand, there will be a second bidder, someone who will (almost certainly) win at a cost of $\sigma^s(\underline{v})$, which produces a benefit of $\alpha\sigma^s(\underline{v})$ to the threshold bidder. Last, with likelihood $C_M^{N-1}F(\underline{v})^{N-M-1}(1-F(\underline{v}))^M$, there will be $M \geq 2$ other active bidders, and with no chance that the threshold bidder will determine the second price, the expected benefits that will accrue to her are $\alpha\sum_{M=2}^{N-1} C_M^{N-1}F(\underline{v})^{N-M-1}(1-F(\underline{v}))^M \int_{\underline{v}}^1 j(x,M)\sigma^s(x)dx$, where, as defined earlier, $j(x,M)$ is the conditional density of the second order statistic. The net benefits of participation are therefore:

$$F(\underline{v})^{N-1}\underline{v} + \alpha(N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))\sigma^s(\underline{v}) + \alpha\sum_{M=2}^{N-1} C_M^{N-1}F(\underline{v})^{N-M-1}(1-F(\underline{v}))^M\int_{\hat{v}}^1 j(x,M)\sigma^s(x)dx - c^s \tag{27}$$

If, on the other hand, the threshold bidder does not participate, she receives 0 with likelihood $F(\underline{v})^{N-1} + (N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))$, the likelihood that one or fewer bids are submitted, since there are no revenue proportional benefits in this case, and $\alpha\int_{\underline{v}}^1 j(x,M)\sigma^s(x)dx$ with likelihood $C_M^{N-1}F(\underline{v})^{N-M-1}(1-F(\underline{v}))^M$ for $M \geq 2$. The net benefits of non-participation are therefore:

$$\alpha\sum_{M=2}^{N-1} C_M^{N-1}F(\underline{v})^{N-M-1}(1-F(\underline{v}))^M\int_{\underline{v}}^1 j(x,M)\sigma^s(x)dx$$

The condition that defines the threshold $\underline{v}$ is therefore:

$$F(\underline{v})^{N-1}\underline{v} + \alpha(N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))\sigma^s(\underline{v}) = c^s \tag{28}$$

the solution of which will be denoted $\underline{v}^s = \underline{v}^s(N, c^s, \alpha)$. Relative to the first price threshold in Eq. (11), two related properties of $\underline{v}^s(N, c^s, \alpha)$ call for attention. First, the threshold is now sensitive to the common return α and warm glow $\gamma = \beta - \alpha$ associated with the charity. Second, when participation costs are the same, $c^f = c^s$, the threshold is lower or, if one prefers, participation rates are higher, in the second price auction. A comparison of the two conditions in Eqs. (11) and (28) reveals that the difference is the term $\alpha(N-1)F(\underline{v})^{N-2}(1-F(\underline{v}))\sigma^s(\underline{v})$, the benefit that accrues to a threshold bidder in second price auction when there is just one other bidder, *and she determines the winner's payment*.

Expected revenues in the second price auction R^s are therefore:

$$\begin{aligned} R^s &= N(N-1)\int_{\underline{v}^s(N,c^s,\alpha,\beta)}^1 F(x)^{N-2}(1-F(x))f(x)\sigma^s(x)\,dx \\ &= \frac{N(N-1)}{1-\gamma}\left(\int_{\underline{v}^s(N,c^s,\alpha,\beta)}^1 F(x)^{N-2}(1-F(x))f(x)x\,dx \right. \\ &\quad \left. + \int_{\underline{v}^s(N,c^s,\alpha,\beta)}^1 F(x)^{N-2}(1-F(x))\int_x^1 (1-F(z))^{\frac{1-\gamma}{\alpha}}dz\,dx\right) \end{aligned} \tag{29}$$

where $N(N-1)F(v)^{N-2}(1-F(v))f(v)$ is the unconditional density function of the second order statistic and the second line follows from substitution for $\sigma^s(x)$.

3. All-pay sealed bid auction

The derivation of the SBNE bid functions under the all-pay mechanism follows now familiar lines. With likelihood $F(\underline{v})^{N-1}$, the representative bidder will have no active rivals, and can expect $(v-(1-\beta)\sigma^a(\hat{v}))$. With likelihood $C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M$, she will have $M \geq 1$ rivals, and expect:

$$\begin{aligned} EU(\hat{v}, v, M) &= \int_{\underline{v}}^{\hat{v}} vg(x,M)dx + \frac{\alpha M}{(1-F(\underline{v}))}\int_{\underline{v}}^1 f(x)\sigma^a(x)dx - (1-\beta)\sigma^a(\hat{v}) \\ &= \frac{(F(\hat{v})-F(\underline{v}))^M}{(1-F(\underline{v}))^M}v + \frac{\alpha M}{(1-F(\underline{v}))}\int_{\underline{v}}^1 f(x)\sigma^a(x)dx - (1-\beta)\sigma^a(\hat{v}). \end{aligned} \tag{30}$$

The first term reflects the fact that she will win the auction, and receive her private value v, with likelihood $G(\hat{v}, M)$. The second and third follow from the observation that, win or lose, she will forfeit the net cost of her bid, $(1-\beta)\sigma^a(\hat{v})$, but obtain benefits equal to a fraction α of the sum of all other bids, expressed here as the product of the number of active bidders M and the mean bid $\int_{\underline{v}}^1 \frac{f(x)}{(1-F(\underline{v}))}\sigma^a(x)dx$. Substitution for $g(x,M)$ in the first term and 6integration then leads to the second line.

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After some simplification, the unconditional payoff $EU(v, \hat{v})$ for a bidder who assumes type $\hat{v}$ is therefore:

$$\begin{aligned} EU(v,\hat{v}) &= F(\underline{v})^{N-1}(v-(1-\beta)\sigma^a(\hat{v})) \\ &+ v\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^M \\ &+ \frac{\alpha}{(1-F(\underline{v}))}\int_{\underline{v}}^1 \sigma^a(x)f(x)dx\textstyle\sum_{M=1}^{N-1} C_M^{N-1}MF(\underline{v})^{N-1-M}(1-F(\underline{v}))^M \\ &-(1-\beta)\sigma^a(\hat{v})\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M \end{aligned} \tag{31}$$

or recalling that $\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M = 1-F(\underline{v})^{N-1}$ and $\sum_{M=1}^{N-1} C_M^{N-1}MF(\underline{v})^{N-1-M}(1-F(\underline{v}))^M = (N-1)(1-F(\underline{v}))$, and then noting that $\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(F(\hat{v})-F(\underline{v}))^M = F(\hat{v})^{N-1} - F(\underline{v})^{N-1}$:

$$\begin{aligned} EU(v,\hat{v}) &= F(\underline{v})^{N-1}(v-(1-\beta)\sigma^a(\hat{v})) + v(F(\hat{v})^{N-1}-F(\underline{v})^{N-1}) \\ &+ \alpha(N-1)\int_{\underline{v}}^1 \sigma^a(x)f(x)dx-(1-\beta)(1-F(\underline{v})^{N-1})\sigma^a(\hat{v}) \\ &= F(\hat{v})^{N-1}v + \alpha(N-1)\int_{\underline{v}}^1 \sigma^a(x)f(x)dx-(1-\beta)\sigma^a(\hat{v}) \end{aligned} \tag{32}$$

The derivative of $EU(v, \hat{v})$ with respect to $\hat{v}$ is therefore just $v(N-1)F(\hat{v})^{N-2}f(\hat{v})-(1-\beta)\frac{d\sigma^a(\hat{v})}{d\hat{v}}$, which equals zero at $\hat{v} = v$ if:

$$\frac{d\sigma^a(v)}{dv} = \frac{N-1}{1-\beta}F(v)^{N-2}f(v)v \tag{33}$$

The solution to this differential equation:

$$\sigma^a(v) = \frac{N-1}{1-\beta}\int F(v)^{N-2}f(v)v + k \tag{34}$$

where k is a constant of integration. Since it is optimal for bidders with threshold values to bid zero, $\sigma^a(\underline{v}) = 0$, this becomes:

$$\sigma^a(v) = \frac{N-1}{1-\beta}\int_{\underline{v}}^v F(x)^{N-2}f(x)xdx \tag{35}$$

or, after integration by parts:

$$\sigma^a(v) = \frac{1}{1-\beta}\left(vF(v)^{N-1}-\underline{v}F(\underline{v})^{N-1}\right)-\frac{1}{1-\beta}\int_{\underline{v}}^v F(x)^{N-1}dx. \tag{36}$$

If the costs of participation in first price and all-pay auctions are the same, then so, too, are the participation thresholds.[11] To show this, recall that with likelihood $F(\underline{v})^{N-1}$, the threshold bidder will be the lone participant, and win a prize worth $\underline{v}$ to her for a bid of 0. With likelihood $C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M$, there will be $M \geq 1$ other bidders, each of whom will submit, in expectation, a bid equal to $\int_{\underline{v}}^1 \sigma^a(x)f(x)\,dx$, which produces a benefit equal to $\alpha M\int_{\underline{v}}^1 \sigma^a(x)f(x)\,dx$ for the threshold bidder. The net benefits of participation are therefore:

$$F(\underline{v})^{N-1}\underline{v} + \alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M M\int_{\underline{v}}^1 \sigma^a(x)f(x)\,dx-c^a \tag{37}$$

The net benefits of non-participation, on the other hand, are:

$$\alpha\textstyle\sum_{M=1}^{N-1} C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M M\int_{\underline{v}}^1 \sigma^a(x)f(x)\,dx \tag{38}$$

since, with likelihood $C_M^{N-1}F(\underline{v})^{N-1-M}(1-F(\underline{v}))^M$, there will be $M \geq 1$ other bidders who produce the same non-exclusive benefit of $\alpha M\int_{\underline{v}}^1 \sigma^a(x)f(x)\,dx$. The threshold is therefore defined by:

$$F(\underline{v})^{N-1}\underline{v} = c^a \tag{39}$$

the solution of which is denoted $\underline{v}^a = \underline{v}^a(N, c^a)$

The same demonstration also shows that expected revenues under the all-pay mechanism are equal to:

$$R^a = N\int_{\underline{v}^a(N,c^a)}^1 \sigma^a(v)f(v)\,dv. \tag{40}$$

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[11] If costs are the same across all three mechanisms, then, participation will be lower in both the first price and all-pay than the second price.

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