Abstract Algebra – Worksheet 3

Definitions:
• A group $\langle G, * \rangle$ is said to be *abelian* if $*$ is commutative.
• We say a group is *finite* if the underlying set contains finitely many elements. We say a group is *infinite* if the underlying set contains infinitely many elements.
• For a finite group G, the *order of* G is the number of elements in G.

8. Provide at least two examples of abelian groups.

9. Refer back to question 5. Identify the finite groups in that question, and for each of these state the order of the group.

10. Provide at least two examples of non-abelian groups. For one of these, prove that the group is non-abelian.

11. Suppose $\langle G, * \rangle$ is a group,with s, t and u in G. Prove or disprove as appropriate: If $s * t = u * s$, then $t = u$.

More Notation: For convenience, instead of using "$*$" to denote the group operation, we often use multiplicative notation as follows:

- In place of $a * b$ write ab.
- Denote an inverse of a (the existence of which is ensured by the group axioms), by a^{-1}.
- Let a^1 denote a, and for $n \in \mathbb{N}$, with $n > 1$ define a^n to be aa^{n-1}.

It is important to note that we have simply introduced some notation; the operation "multiplication" in a group is NOT in general familiar old multiplication. Take care when working in an arbitrary group not to take for granted properties of exponents that are familiar from working with the real numbers. So for example, in the next two problems you may *not* assume that $a^m a^n = a^{m+n}$, nor that $(a^m)^n = a^{mn}$; you'll be proving that soon!

12. If $a \in G$ and $n \in \mathbb{N}$ then both $(a^n)^{-1}$ and $(a^{-1})^n$ have unambiguous interpretations in terms of the definitions already given. Prove that these two are in fact equal.

13. Prove that if G is a group, with a in G, then $(a^{-1})^{-1} = a$.