Math 302 – Permutations Interlude

Recall that S_n is the group of permutations of the set $\{1, 2, 3, ..., n\}$. The operation is function composition. Note that S_n is called the *symmetric group of degree n*.

1. Write the permutation $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 6 & 1 & 5 & 2 \end{bmatrix}$ as a product of disjoint cycles.

2. What is the order of the permutation $(a_1 a_2 ... a_k)$?

3. Write each of the following permutations as a product of disjoint cycles:

   a) (1235)(413)
   b) (413)(1235)
   c) (13256)(23)(46512)
   d) (12)(13)(14)(15)

4. What is the order of each of the following permutations?

   a) (124)(357)
   b) (124)(3567)
   c) (124)(357869)
   d) $(a_1 a_2 ... a_j)(b_1 b_2 ... b_k)$, where all of the a's and b's are distinct.

5. Write the permutation (1357246) as a product of two-cycles (also known as *transpositions*). Then write it as a different product of transpositions.

6. Prove: If a permutation α can be expressed as a product of an even number of transpositions, then every decomposition of α into a product of transpositions must have an even number of them.

7. You've just shown that it makes sense to define an *even* permutation as one that can be expressed as a product of an even number of transpositions. Prove that the set of even permutations in S_n forms a subgroup of S_n. NOTE: This subgroup is called the *alternating group of degree n*, denoted A_n.