Math 302 Worksheet 9

1. a) List all of the distinct subgroups of $\mathbb{Z}_{12}$, and identify all of the generators for each.

   b) Do the same for $\mathbb{Z}_{18}$.

2. a) Let G be a cyclic group of order 12, with generator a. List all of the distinct subgroups of G.

   b) Do the same for a cyclic group H of order 18, with generator b.

The Fundamental Theorem of Cyclic Groups Every subgroup of a cyclic group is cyclic. Moreover, if $| < a > | = n$, then the order of any subgroup of $< a >$ is a divisor of n; and, for each positive divisor k of n, the group $< a >$ has exactly one subgroup of order k, namely $< a^{\frac{n}{k}} >$.

We've already proven the first part; the rest of the proof is deferred.

3. Find an example of a noncyclic group, all of whose subgroups are cyclic.

4. Find a collection of distinct subgroups $< a_1 >, < a_2 >, ..., < a_n >$ of $\mathbb{Z}_{240}$ with the property that $< a_1 > \subset < a_2 > \subset ... \subset < a_n >$ with n as large as possible.

5. Let m and n be elements of the (additive) group $\mathbb{Z}$. Find a generator for the group $< m > \bigcap < n >$.

6. Is every subgroup of $\mathbb{Z}$ cyclic? Why? Let $a \in \mathbb{Z}$. Describe all subgroups of $< a >$.