Abstract Algebra - Worksheet 7

Definition. A group G is *cyclic* if $G =< a >$ for some $a \in G$. In this case, a is called a *generator* of G.

1. Show that $\mathbb{Z}_n$ (under addition) is cyclic for any positive integer n.

2. Let G be a group, and let $a \in G$. Prove that if a has (finite) order n, then

   a) $< a >= \{e, a, a^2, ..., a^{n-1}\}$, and

   b) $a^i = a^j$ if and only if n divides $i - j$.

   (So we now have a connection between the two uses of the word "order" in group theory.)

3. Find all generators of $\mathbb{Z}_6$, $\mathbb{Z}_8$, and $\mathbb{Z}_{20}$. What do you notice about the generators?

4. Suppose that $< a >$, $< b >$, and $< c >$ are cyclic groups of orders 6, 8, and 20 respectively. Find all generators of each of these groups.

5. Let G be a group, and let a be an element of order n in G. Prove that if $a^k = e$, then n divides k.