

Math 302 Worksheet 14

1. Prove that a factor group of a cyclic group is cyclic.
2. Is a factor group of an Abelian group necessarily Abelian? Prove or find a counterexample.
3. **Definition.** A *homomorphism* φ from a group G to a group \bar{G} is a mapping from G into \bar{G} that preserves the group operation; that is, $\varphi(ab) = \varphi(A)\varphi(b)$ for all $a, b \in G$.
 - a) Show that the mapping $A \rightarrow \det(A)$ is a homomorphism from $GL(2, \mathbb{R})$ to \mathbb{R}^* .
 - b) Find a homomorphism $\varphi : S_n \rightarrow \mathbb{Z}_2$.
4. **Definition.** The *kernel* of a homomorphism φ is the set $Ker\varphi = \{g \in G | \varphi(g) = e\}$.
Find the kernel of each of the homomorphisms in exercise 5.
5. Prove that if φ is a homomorphism from G to \bar{G} , then $Ker\varphi$ is a subgroup of G .
6. What is the kernel of an isomorphism?