Math 302 Worksheet 12

Definition Let G be a group, let H be a subgroup of G, and let a be any element of G. Then the set $aH = \{ah | h \in H\}$ is the *left coset of H in G containing a*.

1. Let $G = D_4$, and let $K = \{\rho_0, \rho_2\}$. Compute all of the left cosets of K. What do you notice?

2. Do the same for $G = A_4$ and $H = \{(1), (12)(34), (13)(24), (14)(23)\}$.

3. Prove the following properties of cosets, where G is a group, H is a subgroup of G, and $a, b \in G$:

   a) $a \in aH$;

   b) $aH = H$ if and only if $a \in H$;

   c) $aH = bH$ if and only if $a \in bH$;

   d) $aH = bH$ or $aH \bigcap bH = \varnothing$.

4. Prove **Lagrange's Theorem**: If G is a finite group and H is a subgroup of G, then $|H|$ divides $|G|$.

5. Prove the **Corollary**: In a finite group, the order of each element of the group divides the order of the group.