

Math 302 Worksheet 11

1. Suppose that ϕ is an isomorphism from a group G onto a group \bar{G} . Prove that
 - a) ϕ^{-1} is an isomorphism from \bar{G} onto G .
 - b) G is abelian if and only if \bar{G} is abelian.
 - c) G is cyclic if and only if \bar{G} is cyclic.
 - d) If K is a subgroup of G , then $\phi(K) = \{\phi(k) : k \in K\}$ is a subgroup of \bar{G} .

Definition. An *automorphism* is an isomorphism from a group G onto itself. The set $Aut(G)$ consists of all of the automorphisms of G .

2. Find $Aut(\mathbb{Z}_6)$.
3. Let \mathbb{R}^+ be the group of positive real numbers under multiplication. Show that the mapping $\phi(x) = \sqrt{x}$ is an automorphism of \mathbb{R}^+ .
4. Let $r \in U(n)$. Prove that the mapping $\alpha : \mathbb{Z}_n \rightarrow \mathbb{Z}_n$ defined by $\alpha(s) = sr \pmod{n}$ for all $s \in \mathbb{Z}_n$ is an automorphism.
5. Suppose that $\phi : \mathbb{Z}_{50} \rightarrow \mathbb{Z}_{50}$ is an automorphism with $\phi(11) = 13$. Determine a formula for $\phi(x)$.