Abstract Algebra
Problem Set 7

1. Fill in the blank:
   The group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic if and only if _____.
   Then prove the statement.

2. Let G be a group, and let $a \in G$. Prove that $< a^{-1} >=< a >$.

3. Prove that $\mathbb{Z}_n$ has an even number of generators if $n > 2$.

4. Show that the group of positive rational numbers under multiplication is not cyclic.

5. What are the orders of elements in D_{15}? How many elements have each of those orders?

6. Show that A_8 contains an element of order 15.

7. Let p be a prime. Show that in a cyclic group G of order $p^n - 1$, every element is a pth power (that is, every element can be written in the form g^p for some $g \in G$).

8. Prove that $U(2^n)$ $(n \geq 3)$ is not cyclic.