For this project, you will learn how to play the game Mat-Rix-Toe and then investigate certain properties of the game. In particular, you will be looking for an optimal strategy for each player to win. You will work with a partner (assigned by me) to learn the game and complete the first part of the assignment. Here are the due dates:

- Monday, November 9: Each pair will turn in a paper with responses to Questions 1 and 2.
- Monday, November 16: Each of you will turn in an individual paper with responses to all of the questions. You are welcome to consult with your partner, or anyone else in the class (you’ll probably want to play some more games to get some more insight), but you must list, at the end of your paper, the names of all the people with whom you discussed the paper, with a sentence in each case describing that discussion. (For example, “We played a few games with alternate rules and discussed how our strategies changed.”)

Papers should be typed, using Microsoft Word (with the equation editor) or, if you’d like, LaTeX (which requires time to learn, but I can help). I will evaluate your work using the rubric that appears on the “On Writing Proofs” page of the course website.

Description of the game Mat-Rix-Toe:

The board is a square matrix of size $2 \times 2$ or larger. There are two players, the 1-placer and the 0-placer. First a coin is flipped to decide who goes first. Suppose the 1-placer goes first. The 1-placer starts by putting a 1 as an entry somewhere in the matrix. Then the 0-placer puts a zero somewhere, then the 1-placer goes, etc, back and forth until all the entries in the matrix are filled. If the matrix is nonsingular (invertible), the 1-placer wins. If the matrix is singular (not invertible), the 0-placer wins.

Questions:

1. What is the optimal strategy in the $2 \times 2$ case for each player? Assuming each player implements the optimal strategy, who will win? Does it matter who goes first? Make sure to justify your answers.

2. What is the optimal strategy in the $3 \times 3$ case for each player? Assuming each player implements the optimal strategy, who will win? Does it matter who goes first? Make sure to justify your answers.

3. Suppose the rules were changed so that the 1-placer could put in any number instead of just 1s. Would this change the optimal strategies or general results in the $2 \times 2$ case or the $3 \times 3$ case?

4. Suppose the rules were changed so that the 1-placer was trying to make the matrix singular and the 0-placer was trying to make it nonsingular. What would be the optimal strategies in the $2 \times 2$ case and the $3 \times 3$ case? Would it matter who goes first?
Extra Credit Question: Answer this to get up to five extra credit points.

- Make preliminary findings for the $4 \times 4$ case. Make sure that you are using linear algebra concepts to back up your reasoning!

Score Breakdown:

- First two questions (one paper per pair), due November 9: 25%
- Completed Project (submitted individually), due November 16: 75%

Additional Comments:

- You are welcome to come to my office hours if you need help or have questions. For the first part, you should come with your partner.
- Notice that you and your partner will get the same grade for the first part, so be sure to allow time to check each other’s work before you turn it in.
- Late projects will lose 10 percent each day they are late. Projects turned in over three days late will not be accepted.
- It may help to just play the game for a while and keep track of who wins to get an idea of what an optimal strategy would be.

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