Affirmative Action in a Two Stage Statistical Discrimination Model

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November 19, 2009

Abstract

We investigate the effect of affirmative action policies in a statistical discrimination model with two stages. A group of workers face discrimination in both hiring and promotion decisions of the firm, and the regulator is free to intervene in both stages. We show that there are affirmative action policies which can eliminate negative employer beliefs by increasing educational investment in the discriminated group. Further, affirmative action imposes a disincentive effect on educational investment only for policies which require the firm to promote workers who are clearly unqualified. Our results show that while it is always beneficial to intervene in the hiring stage, the welfare effect of promotion stage intervention is ambiguous. We subsequently characterize the optimal affirmative action plan and show that the optimal policy may require intervention in both stages. Lastly, we consider the feasibility of introducing affirmative action policies, in terms of ensuring compliance by the firm. We show that arbitrary policies may indeed violate the participation constraint of the firm. Nevertheless, the optimal policy is always feasible, since it allows the firm positive profit.

Keywords: affirmative action, statistical discrimination, stereotypes.
JEL Classification Numbers: J71, J78

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This project was initiated under the guidance of Subir Bose. I am also grateful to Mamoru Kaneko, Dan Bernhardt, and Robert Prasch for valuable advice and criticism. While I take this opportunity to thank them, all errors are entirely mine.

1. Introduction

Can affirmative action policies improve the labor market prospects of a discriminated group? There is an extensive literature in economics which addresses this question, employing a variety of theoretical and empirical methods. Statistical discrimination models assume that groups are ex ante identical and firms have no prior preference for one group over another. However, worker productivity is not directly observable, so that firms rely on variables imperfectly correlated with productivity in order to evaluate the expected productivity of a job applicant. In particular, firms may use the group identity of a worker to infer his expected productivity. In this situation, discrimination arises because firms believe that one group of workers are less likely to be qualified than another, and these negative beliefs come to form a self-fulfilling prophecy in equilibrium.

It has been argued in this context that affirmative action in the form of a policy that seeks to equalize the labor market presence of all groups may fail to eliminate the negative *stereotypes* held by firms (Coate and Loury (1993) [8]). For example, consider the hiring decision of a firm. If the firm believes that workers from a group are less likely to be qualified, it will meet the equal representation constraint by lowering the standard used in hiring workers from this group. This may reduce the incentive for educational investment in the discriminated group, since workers now find it easier to get hired. Hence, negative employer beliefs may persist in the face of policy intervention.

Most contributions to the statistical discrimination literature model affirmative action as a requirement that the firm should hire a specified fraction of job applicants from the discriminated group (see Fryer *et al* (2008) [9] or Fryer and Loury (2005) [12] as exceptions). In the sense that the labor market prospects of a worker depend on his group identity even under policy intervention, affirmative action in most statistical discrimination models is *group sighted*. In contrast, this paper looks at affirmative action policies which are independent of group identity; and depend solely on observed signals of productivity, such as college transcripts or scores on standardized tests and job entrance examinations.

The basic idea we pursue is simply this: if policy intervention compels the firm to ignore group identity altogether and infer the expected productivity of a job applicant solely from his labor market signal, can this eliminate negative prior beliefs? In other words, if policy intervention compels the firm to treat workers with the same test score identically regardless of their group identities, would this equalize the incentive for skill acquisition across groups and hence, lead to the equality of employer beliefs? We show that while arbitrary policies of this type may indeed have ambiguous welfare effects, the optimal policy will successfully eliminate negative stereotypes held by firms.

Most of the theoretical literature on affirmative action looks at discrimination in a single labor market decision, in the sense that a group is discriminated either in the hiring or in the subsequent promotion decision of the firm. However, a worker may face discrimination in both stages of his career, and his decision to invest in education is affected both by the possibility of entry level discrimination and by the possibility of meeting a *glass ceiling*, once he successfully overcomes the initial adversity.[1] Hence, focusing on discrimination in a single stage may not completely capture the interdependence of negative employer beliefs and investment incentives in a discriminated group.

Secondly, the discriminatory behavior of the firm in any one stage may not be independent of that in the other stage, since the firm can acquire information about a group over time. If the firm believes that workers from a group will not do well in senior positions, however satisfactorily they perform in entry level jobs; it may be inclined to discriminate against this group in hiring decisions as well. Conversely, and as pointed out by Fryer (2006) [10], workers who successfully overcome discrimination in the entry level, may face less discrimination in being promoted, if the firm takes into account the fact that these workers faced a more stringent hiring standard and are consequently more qualified on the average than workers hired from the non-discriminated group.

Further, if a worker faces discrimination in both stages of his career, this raises an important question as to the optimal timing of policy response. Should the regulator intervene in any one or both stages, if society is to avoid the unintended consequences of affirmative action in propagating negative stereotypes? We address these concerns in a statistical discrimination model with sequential hiring and promotion decisions, and consider the role of affirmative action policies based on observed labor market signals of workers, rather than a targeted increase in the occupational attainment of the discriminated group. We characterize the optimal affirmative action plan when policy response takes the form indicated above, and show that the optimal policy can eliminate negative stereotypes in both labor market decisions. Our analysis is sensitive to the fact that the effect of educational investment may differ over the two stages in the career of a worker. The model described in Section 2 considers a situation where early investment allows a worker to reap benefits in subsequent stages of his career. Hence, the optimal policy gives more incentive for investing in the hiring stage than investing in the promotion stage.

The last question we address pertains to the feasibility of policy response to discrimination. In other words, would the firm continue to operate under an affirmative action policy or would the expected cost imposed by the policy in terms of requiring the firm to hire and promote unqualified workers compel it to shut down? We show that arbitrary policies of the type considered in this paper may indeed violate the participation constraint of the firm. However, the optimal affirmative action plan is unambiguously feasible, as it allows the firm positive profit.

[1] The term *glass ceiling* refers to "...the severe under-representation of females and minorities at the highest levels of occupational achievement throughout a wide range of occupations.."(Bjerk (2007) [4], page2).

Related Literature: The statistical discrimination literature originates in the work of Arrow (1973) [1] and Phelps (1972) [16]. Both models are based on the idea that worker productivity is imperfectly observable. Hence, firms rely on group identity to infer the productivity of a job applicant. Phelps [16] assumed that minorities emit a less informative labor market signal. In contrast, Arrow [1] showed that statistical discrimination can occur even when groups are ex ante identical: if firms hold negative prior beliefs that workers from the minority group are less likely to be qualified than workers from the majority, they are less likely to hire the former. Anticipating discrimination in the labor market, minority workers thus have less incentive to invest in education than workers from the majority, and the negative prior beliefs of the firm come to be confirmed in equilibrium.

As mentioned earlier, an important contribution to this literature is Coate and Loury (1993) [8], who show that affirmative action in the form of an equal representation constraint may fail to eliminate negative prior beliefs on the part of the firm. This paper has originated a substantial theoretical literature on affirmative action, framed in the context of statistical discrimination models.[2] While a complete review of this literature is beyond the scope of this paper (see Fryer and Loury (2005a) [11]), we should mention the contributions by Moro and Norman (2003, 2004) [15], [14], which extend the partial equilibrium model of Coate and Loury [8] to allow variable wages. They show that in a general equilibrium environment, affirmative action will unambiguously eliminate negative stereotypes, even though the discriminated group may be worse off than before due to increasing income inequality.

The contribution closest to our own is Fryer (2006) [10], who proposes a two stage model of discrimination with sequential hiring and promotion decisions. Indeed, this is the basic model used in our analysis of affirmative action. In contrast to our focus on policy intervention, however, Fryer [10] looks at circumstances under which a firm discriminating against one group in hiring decisions, may switch to discriminating against the other in promotion decisions - a phenomenon he calls *belief-flipping*. A related contribution is Bjerk (2007) [4], who focuses on the glass ceiling phenomenon in a multistage model of the labor market. However, skill investment decisions are exogenous in his model; and discrimination arises from the assumption that groups differ in average skill levels, precision of labor market signals, and opportunities to signal job skills to the employer.

We end this review by citing three branches of literature which are of related interest. There is an extensive theoretical literature on affirmative action at the education level (see Chan and Eyster (2003) [5] for example), based both on statistical discrimination and other approaches. Secondly, there is a class of models which look at the benefits

[2] There is a second branch of literature, which introduces affirmative action policies in search-matching models of the labor market (see Rosen (2003) [17] for example). However, these models are based on the assumption that employers have a conscious preference for one group of workers over another.

from affirmative action in terms of creating role models or mentors in a disadvantaged group (see Chung (2000) [7] and Athey, Avery and Zemsky (2000) [2]). Lastly, there is an emerging literature which introduces peer or neighborhood effects into the canonical statistical discrimination model (see Chaudhuri and Sethi (2007) [6] for example).

Structure of the Paper: Section 2 describes the basic model and defines the notion of equilibrium. Section 3 investigates the impact of affirmative action on educational investment in the hiring and promotion stages. Section 4 investigates the feasibility of introducing affirmative action policies in the sense of ensuring compliance by the firm. Section 5 brings us to the main focus of this paper, namely, the characterization of the optimal affirmative action plan. Section 6 provides a discussion of our model and indicates avenues for future research, while Section 7 concludes by providing a brief summary of our analysis. Proofs omitted from the main text are collected in the Appendix.

2. A Model of Statistical Discrimination in Two Stages

The basic model is from Fryer (2006) [10] and we provide a brief description of its components in Section 2.1, retaining the original notation wherever possible. Section 2.2 defines the notion of equilibrium in this model and exhibits four types of equilibria which play a role in the subsequent analysis of affirmative action.

2.1. Components of the Model

Players and Sequence of Moves: There is a single firm and two groups of workers A and B, each of unit mass.[3] One of the two groups is assumed to face discrimination from the firm in the hiring (h) and promotion (p) stages. The timing of the game is presented in Fig. 2.1.

Stage 0: Prior to the actual play of the game, nature assigns a type c to each worker, where c denotes the cost of educational investment in each stage $j \in \{h, p\}$. Thus, the cost of investing in education is assumed to be the same in the two stages. Following Fryer [10], we also assume that c is distributed uniformly on $[0, 1]$ for both groups. The type of a worker is private information for him.

Stage 1 (Hiring Stage): At the beginning of the hiring stage, each worker observes his cost of educational investment c.

(1.1): Each worker makes a binary decision on whether or not to invest in education.

[3] The assumption of a single fim is for analytical convenience only. We could easily assume a continuum of firms with measure one. In fact, the scenario we have in mind is an economy with a large number of firms and workers, where workers are randomly matched with firms. Since no individual firm can deviate profitably from a discriminatory equilibrium, there is a role for policy intervention.

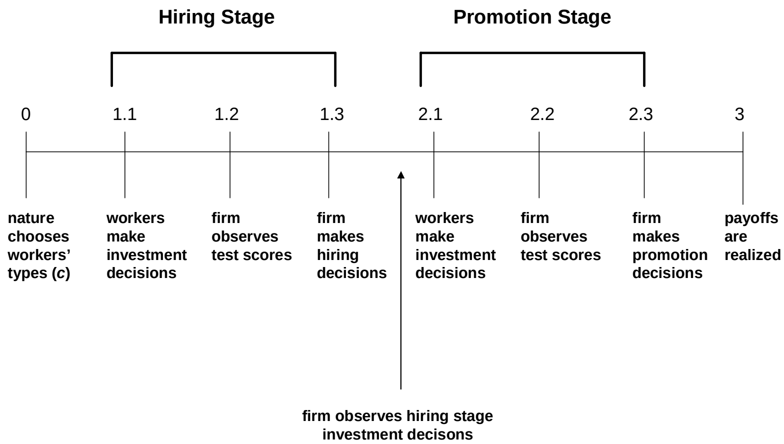

Figure 2.1: Sequence of Moves

We assume that he becomes qualified for the entry level job in the firm if he invests and remains unqualified if he does not. Hence, there is no uncertainty as regards the outcome of educational investment. For each worker, nature then emits a signal on his investment decision. The signal can be interpreted as the score of a worker in a job entrance examination. It takes a value from $\{pass, unclear\}$ if the worker has invested and from $\{unclear, fail\}$ if he has not. Let p_q (p_u) be the probability that a worker draws the unclear signal when he is qualified (unqualified). We assume that $p_q > p_u$ and $p_q + p_u < 1$. Note that these assumptions, in essence, restrict the testing technology. The first assumption implies that a qualified worker is more likely to receive an unclear signal than an unqualified worker and the second assumption implies that the probability of receiving an unclear signal is not too high for either type of worker.

(1.2): The firm does not observe the investment decision of a worker but receives his hiring stage signal. Note that the firm knows for sure that a worker with the *pass* signal is qualified and one with the $fail$ signal is unqualified. Hence, the hiring stage signal is perfectly informative, except in the case where a worker draws the *unclear* signal.

(1.3): The firm takes a binary decision on whether to hire or reject a worker, based on his hiring stage signal or entrance test score. If a worker is rejected, he exits the game forever. If he is hired, he is employed in the entry level job.

Following Fryer [10], we assume that the firm observes the investment decisions of the workers between the hiring and the promotion stages. If a worker has not invested in the hiring stage, he is fired and exits the game forever. If he has invested, he becomes eligible for the promotion stage.[4]

Stage 2 (Promotion Stage): The decision structure in the promotion stage is specified analogously.

(2.1): A worker again takes a binary investment decision, choosing whether or not to be qualified for the promotion level job. Note that promotion stage investment can be regarded as on-job training needed to fulfill the responsibilities of the upper level job.

(2.2): As before, nature emits a noisy signal on the investment decision of a worker, distributed identically as the hiring stage signal and potentially interpreted as his score in a promotion examination.

(2.3): Based on the observed promotion stage signal, the firm again takes a binary decision on whether to promote a worker or keep him in the entry level job.

Stage 3: The game ends with payoffs being realized for all players.

Strategies: In each stage, a worker decides whether or not to invest, based on his cost c. The only cost type indifferent between investing and not investing is of measure zero, since we have assumed a continuous distribution for c. Hence, we can focus on pure strategies without loss of generality. A hiring stage strategy for a group i worker is thus a function of the form $[0,1] \mapsto \{invest, not\ invest\}$. Recall that no worker is eligible for promotion if he has not invested in the hiring stage. Hence, a promotion stage strategy for a group i worker is a function $\{invest, not\ invest\} \times [0,1] \mapsto \{invest, not\ invest\}$, where the first term in the Cartesian product represents hiring stage investment.

The firm decides to hire a worker from group i based on his group identity $i \in \{A, B\}$ and his hiring stage signal. Hence, a hiring stage strategy for the firm is a function $\{A, B\} \times \{pass, unclear, fail\} \mapsto \{hire, not\ hire\}$. In the promotion stage, however, the decision to promote a group i worker depends on whether he has invested in the hiring stage, in addition to his group identity and promotion stage signal. Hence, a promotion stage strategy for the firm is a function of the form $\{invest, not\ invest\} \times \{A, B\} \times \{pass, unclear, fail\} \mapsto \{promote, not\ promote\}$.

Payoffs: Players earn payoffs in both the hiring and the promotion stage and the total payoff of a player is the undiscounted sum of the stage payoffs. A worker who is not hired exits the game with payoff 0. A worker who is hired gets $1 - c$ if he invested in

[4] Ideally, what one would like to capture is a situation where investment in the early stage of a worker's career changes the cost of subsequent investment. To keep the analysis tractable, the present model considers an extreme version of this story. It is assumed that if a worker does not invest in the hiring stage, the cost of promotion stage investment is prohibitively high.

the hiring stage and 1 if he did not. The firm gets 0, both if it rejects a worker and if it hires a qualified worker. If it hires an unqualified worker, it gets -1.

In the promotion stage, a promoted worker gets $\lambda - c$ if he invested in the promotion stage and λ if he did not. A worker who is not promoted remains in the entry level job and gets 1, provided he invested in the hiring stage.[5] The firm gets $\alpha - \lambda \geq 0$ if it promotes a qualified worker and $-\lambda$ if it does not. If it does not promote a worker who invested in the hiring stage, it again gets 0.

2.2. Equilibrium

Since the signal of a worker is perfectly informative about his qualifications when he draws the pass or fail signal, the optimal decision of the firm will require it accept pass signals and reject fail signals with certainty.[6] Hence, we can think of equilibrium firm strategies in terms of the decision to hire or promote an unclear signal. For $i = A, B$ and $j \in \{h, p\}$, let π_i^j be the prior belief of the firm that a worker with unclear signal from group i is qualified for the stage j job. Let $\mu_i^j \in [0, 1]$ denote the probability that the firm hires or promotes a worker from group i with unclear signal. Since μ_i^j depends on π_i^j, the equilibrium firm strategy can be denoted as $(\mu_i^{h*}(\pi_i^h, \pi_i^p), \mu_i^{p*}(\pi_i^h, \pi_i^p))$ for $i = A, B$.

Without loss of generality, the equilibrium investment decision of any worker can be represented by a pair of threshold strategies $(c_i^{h*}(\mu_i^h, \mu_i^p), c_i^{p*}(\mu_i^p)) \in [0, 1]^2$ where a group i worker invests in the hiring stage if and only if $c \leq c_i^{h*}$, and conditional on having invested in the hiring stage and being hired by the firm, invests in the promotion stage if and only if $c \leq c_i^{p*}$. The arguments of c_i^{h*} and c_i^{p*} reflect the fact that the investment cost thresholds are group-specific and depend on the probability μ_i^j with which the firm promotes unclear signals from group i.

For $i = A, B$ and $j \in \{h, p\}$, we define mappings $\Phi_i^j : [0, 1]^2 \to [0, 1]$ such that

$$\Phi_i^h(\pi_i^h, \pi_i^p) = c^{h*}(\mu_i^h(\pi_i^h, \pi_i^p), \mu_i^{p*}(\pi_i^h, \pi_i^p))$$

and

$$\Phi_i^p(\pi_i^h, \pi_i^p) = \frac{c^{p*}(\mu_i^{p*}(\pi_i^h, \pi_i^p))}{c^{h*}(\mu_i^h(\pi_i^h, \pi_i^p), \mu_i^{p*}(\pi_i^h, \pi_i^p))}.$$

$\Phi_i^j(\pi_i^1, \pi_i^2)$ is interpreted as the fraction of group i workers investing in stage j under the best responses of all players, given beliefs (π_i^1, π_i^2) on the part of the firm. Equilibrium

[5] Recall that the firm observes hiring stage investment decisions before the promotion stage and fires workers who did not invest.

[6] The firm knows that a worker with the pass signal is qualified. Hence, the expected payoff from promoting him is $\alpha - \lambda \geq 0$ while the expected payoff from not promoting him is 0. In the hiring stage, the firm is indifferent between hiring and rejecting a pass signal, as the payoff in either case is 0.

Type of Equilibrium	Hiring Stage Beliefs	Promotion Stage Beliefs
C-C	$2(1-p_q)$	$\frac{\lambda-1}{2}$
C-L	$(1-p_q)[2+p_u(\lambda-1)]$	$\frac{(1-p_u)(\lambda-1)}{(1-p_q)[2+p_u(\lambda-1)]}$
L-C	1	$(1-p_q)(\lambda-1)$
L-L	1	$(1-p_u)(\lambda-1)$

Figure 2.2: Hiring and Promotion Stage Beliefs in Different Equilibria

of the game is defined as follows:

Definition 2.1 (Equilibrium): An equilibrium is a vector $((\pi_A^{h*}, \pi_A^{p*}), (\pi_B^{h*}, \pi_B^{p*}))$ of beliefs satisfying for each $i = A, B$,

$$\pi_i^{h*} = \Phi_i^h(\pi_i^h, \pi_i^p) \text{ and } \pi_i^{p*} = \Phi_i^p(\pi_i^h, \pi_i^p).$$

In other words, equilibrium is a pair of *self-confirming* hiring and promotion stage beliefs for each group. We refer the reader to Fang and Fryer (2003) [13] for a discussion of the existence issue, and focus on four types of equilibria which play an important role in the subsequent policy analysis.

The firm is said to be *liberal* towards group i in stage j if it sets $\mu_i^j = 1$ and *conservative* if it sets $\mu_i^j = 0$. Fryer [10] identifies the following types of equilibria with respect to a given group: (1) Conservative-Conservative $[(\mu^h, \mu^p) = (0,0)]$; (2) Conservative-Liberal $[(\mu^h, \mu^p) = (0,1)]$; (3) Liberal-Conservative $[(\mu^h, \mu^p) = (1,0)]$; and (4) Liberal-Liberal $[(\mu^h, \mu^p) = (1,1)]$. Figure 2.2 summarizes the hiring and promotion stage beliefs of the firm under the four types of equilibria. A detailed description of the Conservative-Conservative (C-C) equilibrium is provided in the next section and the reader is referred to Fryer [10] for an analysis of the other three types of equilibria.

3. Affirmative Action Policies

This section introduces affirmative action policies into the model presented in Section 2. An affirmative action policy in stage $j \in \{h, p\}$ is defined as a function

$$\psi^j : \{pass, unclear, fail\} \rightarrow [0, 1].$$

That is, for each test score, a policy sets the firm a specific probability with which to hire or promote that worker. For convenience, we use the notation $\psi^j(unclear) = \psi^j_U$ and $\psi^j(fail) = \psi^j_F$.

Recall that in any equilibrium, the firm will hire or promote pass signals with probability one. Similarly, fail signals are rejected or denied promotion with certainty. Depending upon the treatment of unclear and fail signals, we, therefore, distinguish between two types of policies:

(I): *Type I* policies require the firm to hire or promote unclear signals with probability $\psi^j_U \in [0, 1]$.

(II): *Type II* policies require the firm to hire or promote unclear signals for sure, and fail signals with probability $\psi^j_F \in [0, 1]$.[7]

The remainder of this section looks at the impact of affirmative action on negative stereotypes held by the firm. Since workers with the same labor market signal are treated identically under the policies defined above, we focus on the discriminated group without loss of generality. To place the consequences of affirmative action in starkest contrast with the situation prior to policy intervention, we assume the discriminated group to be in the C-C equilibrium when the regulator decides to intervene. For the convenience of the reader, Section 3.1 presents the C-C equilibrium of Fryer [10] in detail. Sections 3.2 and 3.3 then consider the impact of hiring and promotion stage intervention respectively, while Section 3.4 concludes by looking at the impact of simultaneous intervention in the two stages.

3.1. The C-C Equilibrium

Following standard procedure, we solve the game backwards, starting from the decision of the firm to promote a worker or keep him at the entry level job.

Promotion Decision of Firm: As noted before, a pass signal is promoted and a fail signal is rejected with certainty in any equilibrium of the game. Hence, we focus on

[7] One can think of policies which require the firm to hire unclear signals with probability less than one, at the same time as hiring fail signals with positive probability. These policies cannot be optimal: relative to a Type II policy, the firm cannot gain by rejecting workers who may be qualified and hiring workers who are unqualified for sure. The same argument applies to promotion decisions.

the promotion decision for a worker with unclear signal. Given the prior belief π^p, the posterior probability assigned to the event that a worker with unclear signal is qualified for the promotion stage is

$$\xi(\pi^p, unclear) = \frac{\pi^p p_q}{\pi^p p_q + (1-\pi^p)p_u}. \tag{3.1}$$

The expected payoff from promoting a worker with unclear signal is

$$\xi(\pi^p, unclear)(\alpha - \lambda) - [1 - \xi(\pi^p, unclear)]\lambda$$

and the expected payoff from not promoting him is 0. Hence, the belief π^p at which the firm is indifferent between promoting and not promoting a worker with unclear signal is obtained as

$$\hat{\pi}^p = \frac{1}{1 + \frac{p_q}{p_u}(\alpha - \lambda)\frac{1}{\lambda}}.$$

π^p is interpreted as the maximum belief that supports the conservative promotion policy.

Promotion Stage Investment Decision of worker: The promotion stage investment decision of a worker will depend on the probability with which he believes the firm to promote unclear signals, that is, μ^p. In the C-C equilibrium, workers correctly believe $\mu^p = 0$, so that the expected payoff from investing is $(1-p_q)\lambda + p_q - c$ and the expected payoff from not investing is 1. Hence, the promotion stage investment threshold is

$$c^{p*}(\mu^p = 0) = c_0^{p*} = (1-p_q)(\lambda - 1).$$

Since c is distributed uniformly on $[0,1]$, c_0^{p*} is the fraction of workers investing in the promotion stage of a C-C equilibrium.

Hiring Decision of Firm: If a worker is not qualified for the entry level job, the expected payoff from hiring him is $-1 + 0 = -1$. On the other hand, if he is qualified, then the expected payoff from hiring him is $0 + V(\pi^p) = V(\pi^p)$, where $V(\pi^p)$ is the ex ante expected promotion stage payoff of the firm, that is, the promotion stage payoff calculated prior to observing the promotion stage signal.

The firm assigns probability π^p to the event that the worker will invest for promotion. In this case, the expected promotion stage payoff of the firm is $\pi^p(1-p_q)(\alpha - \lambda)$. On the other hand, it it assigns probability $(1-\pi^p)$ to the event that the worker will not invest for promotion and the expected payoff in this case is 0. Hence,

$$V(\pi^p) = \pi^p(1-p_q)(\alpha - \lambda).$$

Given the prior belief π^h that a worker with unclear signal is qualified for the entry level job, the posterior probability assigned to this event is $\xi(\pi^h, unclear)$, where

$\xi(\pi^h, unclear)$ is defined in the same way as (3.1). Hence, the expected payoff from hiring a worker with unclear signal is

$$\xi(\pi^h, unclear)V(\pi^p) - [1 - \psi(\pi^h, unclear)].$$

The expected payoff from rejecting a worker is 0. Hence, the belief at which the firm is indifferent between hiring an unclear signal and rejecting him is

$$\hat{\pi}_0^h = \frac{1}{1 + \frac{p_q}{p_u}[\pi^p(1 - p_q)(\alpha - \lambda)]}. \tag{3.2}$$

Analogous to π^p, $\hat{\pi}_0^h$ is interpreted as the minimum belief which supports the liberal hiring policy. The subscript 0 refers to the fact that it has been calculated on the basis that the firm is conservative in the promotion stage, that is sets $\mu^p = 0$.

Hiring Stage Investment Decision of Worker: Workers correctly believe that $\mu^h = 0$. Hence, the expected payoff to a worker who invests is $(1 - p_q)[1 + R_0(c)] - c$, where $R_0(c)$ is his ex ante expected promotion stage payoff, given cost of promotion stage investment c and conservative promotion policy $\mu^p = 0$. However, the expected payoff to a worker who does not invest is 0. Hence, the promotion stage investment threshold is $c^{h*}(\mu^h = 0, \mu^p = 0) = c_{0,0}^{h*} = (1 - p_q)[1 + R_0(c)]$.

To obtain $R_0(c)$, recall that the expected payoff from not investing in the promotion stage is $(1 - p_q)\lambda + p_q - c$ while the expected payoff from not investing is 1. Hence,

$$R_0(c) = \max\{(1 - p_q)\lambda + p_q - c; 1\} = 1,$$

so that $c_{0,0}^{h*} = 2(1 - p_q)$. Again since c is distributed uniformly on $[0, 1]$, $c_{0,0}^{h*}$ denotes the fraction of workers investing in the hiring stage in the C-C equilibrium.

Equilibrium Hiring and Promotion Stage Beliefs: Recall from Definition 2.1 that an equilibrium stage j belief of the firm is such that in choosing the best response to that belief, the firm induces a level of stage j investment from the workers which makes the belief correct. Hence, the hiring stage belief in the C-C equilibrium is given by

$$\pi_{0,0}^{h*} = c_{0,0}^{h*} = 2(1 - p_q)$$

and the promotion stage belief is given by

$$\pi_{0,0}^{p*} = \frac{c_0^{p*}}{c_{0,0}^{h*}} = \frac{(1 - p_q)(\lambda - 1)}{2(1 - p_q)} = \frac{\lambda - 1}{2}.$$

For the subsequent policy analysis, we focus on the case where $\pi_{0,0}^{h*}, \pi_{0,0}^{p*} < 1$. This imposes the restrictions $p_q < \frac{1}{2}$ and $\lambda < 3$.

3.2. Hiring Stage Intervention

Affirmative action in the hiring stage will not affect the incentive to invest in the promotion stage. Hence, the promotion stage investment threshold c_0^{p*} remains unchanged from Section 2, as does the ex ante expected promotion stage payoff of a worker $R_0(c)$. We first consider the effect of Type I policies on the incentive to invest in the hiring stage.

Type I Policies in the Hiring Stage: As mentioned before, a Type I policy requires the firm to hire unclear signals with probability $\psi_U^h \in [0,1]$. The expected payoff of a worker who invests in the hiring stage is

$$[(1-p_q)+p_q\psi_U^h][1+R_0(c)]-c = 2[(1-p_q)+p_q\psi_U^h]-c.$$

However, the expected payoff of a worker who does not invest in the hiring stage is $p_u\psi_U^h$. With a slight abuse of notation, the hiring stage investment threshold is denoted by

$$c_{0,0}^{h*}(\psi_U^h) = 2(1-p_q)+(2p_q-p_u)\psi_U^h. \tag{3.3}$$

Since $\frac{\partial}{\partial\psi_U^h}[c_{0,0}^{h*}(\psi_U^h)] = 2p_q - p_u > 0$, the expected fraction of workers investing in the hiring stage increases in the probability with which the firm is required to hire unclear signals.

Recall that $c_{0,0}^{h*} = 2(1-p_q) < 1$. Since $c_{0,0}^{h*}(\psi_U^h = 1) = 2-p_u > 1$ and $c_{0,0}^{h*}(\psi_U^h)$ is monotonically increasing in ψ_U^h, there is $\psi_U^h \in (0,1)$ such that $c_{0,0}^{h*}(\psi_U^h) = 1$. Using (3.3), ψ_U^h is obtained as $\frac{2p_q-1}{2p_q-p_u}$. This suggests that any policy which requires the firm to hire unclear signals with probability greater or equal to ψ_U^hwill make all workers invest in the hiring stage. Hence, all Type II policies have this effect.

Type II Policies in the Hiring Stage: A Type II policy requires the firm to hire unclear signals with probability 1 and fail signals with probability $\psi_F^h \in [0,1]$. The expected payoff of a worker who invests in the hiring stage is $[1+R_0(c)]-c = 2-c$ and the expected payoff of a worker who does not invest is $p_u+(1-p_u)\psi_F^h$. Hence, the hiring stage investment threshold under the policy is obtained as

$$c_{0,0}^{h*}(\psi_F^h) = 2-p_u-(1-p_u)p_{\Delta^H}^F.$$

We see that any Type II policy will make all workers invest in the hiring stage, since $\min_{\psi_F^h} c_{0,0}^{h*}(\psi_F^h) = c_{0,0}^{h*}(\psi_F^h = 1) = 1$. Note also that any equilibrium under Type II policies will see the firm hire all workers: any Type II policy requires the firm to hire unclear signals with probability one. Hence, all workers invest in the hiring stage and the only signals observed are pass and unclear. All pass signals are hired in any equilibrium of the game. By hiring all unclear signals, the firm, in effect, hires the entire population.

We conclude the discussion on hiring stage policies by summarizing our observations from the preceding analysis. We have:

Fact 3.1 (Impact of Hiring Stage Policies): Consider the C-C equilibrium. Then (1): any Type I policy in the hiring stage with $\psi_U^h \geq \frac{2p_q-1}{2p_q-p_u}$ will make all workers invest for the entry level job;

(2): any Type II policy in the hiring stage will make all workers invest for the entry level job; and

(3): any hiring stage policy with $\psi_U^h = 1$ will have the firm hiring all workers in equilibrium.

3.3. Promotion stage Intervention

In contrast to hiring stage intervention, affirmative action in the promotion stage affects the incentive to invest in both stages of the game and the promotion stage investment threshold will differ according to the type of policy being introduced. As before, we first consider the introduction of Type I policies in the promotion stage.

Type I Policies in the Promotion Stage: In this case, the firm is required to promote unclear signals with probability $\psi_U^p \in [0,1]$. The expected payoff of a worker who invests in the promotion stage is now given by $(1-p_q)\lambda + p_q[\psi_U^p\lambda + (1-\psi_U^p)] - c$ and the expected payoff of a worker who does not invest is $(1-p_u)+p_u[\psi_U^p\lambda+(1-\psi_U^p)]$. The promotion stage investment threshold is thus obtained as

$$c_0^{p*}(\psi_U^p) = (1-p_q)(\lambda-1) + (p_q-p_u)(\lambda-1)\psi_U^p. \tag{3.4}$$

Since $p_q > p_u$, we have $\frac{\partial}{\partial \psi_U^p}[c_0^{p*}(\psi_U^p)] = (p_q-p_u)(\lambda-1) > 0$. That is, the expected fraction of investors increases in the probability with which the firm is required to promote unclear signals.

We have $\max\limits_{\psi_U^p} c_0^{p*}(\psi_U^p) = c_0^{p*}(\psi_U^p = 1) = (1-p_u)(\lambda-1)$, which is equal to the level of promotion stage investment in the L-L equilibrium. Recall from Section 2 that the hiring and promotion stage beliefs in the C-L equilibrium are given by $\pi_{0,1}^{h*} = (1-p_q)[2+p_u(\lambda-1)]$ and $\pi_{0,1}^{p*} = \frac{(1-p_u)(\lambda-1)}{(1-p_q)[2+p_u(\lambda-1)]}$ respectively. Since $\pi_{0,1}^{j*} < 1$, we have $\max\limits_{\psi_U^p} c_0^{p*}(\psi_U^p) < 1$. Thus, no Type I policy in the promotion stage can make all workers invest for promotion.

For Type I policies, the ex ante expected promotion stage payoff is given by

$$\begin{aligned} R_0(c,\psi_U^p) &= \max\{(1-p_q)\lambda + p_q[1+\psi_U^p(\lambda-1)] - c, 1 + p_u\psi_U^p(\lambda-1)\} \\ &= 1 + p_u\psi_U^p(\lambda-1). \end{aligned} \tag{3.5}$$

Changed expectations about the promotion stage payoff will affect the hiring stage investment decision. Using (3.5), the expected payoff of a worker who invests is now given by $(1-p_q)[1+R_0(c,\psi_U^p)]-c=(1-p_q)[2+p_u\psi_U^p(\lambda-1)]$. On the other hand, the expected payoff of a worker who does not invest is 0. Hence, the hiring stage investment threshold is obtained as

$$c_{0,0}^{h*}(\psi_U^p)=(1-p_q)[2+p_u\psi_U^p(\lambda-1)].$$

Since $\frac{\partial}{\partial\psi_U^p}[c_{0,0}^{h*}(\psi_U^p)]=p_u(1-p_q)(\lambda-1)>0$, we see that expected hiring stage investment increases in the strength of the policy being implemented.

We have $\max_{\psi_U^p} c_{0,0}^{h*}(\psi_U^p)=c_{0,0}^{h*}(\psi_U^p=1)=(1-p_q)[2+p_u(\lambda-1)]<1$. Recalling that $\max_{\psi_U^h} c_{0,0}^{h*}(\psi_U^h)=2-p_u>1$ and that $c_{0,0}^{h*}(.)$ is monotonically increasing in ψ_U^h and ψ_U^p, we have for any $p\in[0,1]$,

$$c_{0,0}^{h*}(p=\psi_U^h)\geq c_{0,0}^{h*}(p=\psi_U^p). \tag{3.6}$$

This suggests that any Type I policy will stimulate greater hiring stage investment if it is introduced in the hiring stage than if it is introduced in the promotion stage.

Type II Policies in the Promotion Stage: Here, the expected payoff of a worker who invests in the promotion stage is $\lambda-c$, while his expected payoff from not investing is $p_u\lambda+(1-p_u)[1+\psi_F^p(\lambda-1)]$. Hence, the promotion stage investment threshold is given by

$$c_0^{p*}(\psi_F^p)=(1-p_u)(\lambda-1)(1-\psi_F^p). \tag{3.7}$$

We have $\frac{\partial}{\partial\psi_F^p}[c_0^{p*}(\psi_F^p)]=-(1-p_u)(\lambda-1)<0$ and $c_0^{p*}(\psi_F^p=1)=0$, so that the expected fraction of workers investing in the promotion stage is decreasing, until it falls to zero when the firm promotes all fail signals for sure. Since $\max_{\psi_F^p} c_0^{p*}(\psi_F^p)=(1-p_u)(\lambda-1)<1$, it follows that there is no Type II policy which can make all workers to invest in the promotion stage.

For Type II policies, the ex ante expected promotion stage payoff is given by

$$\begin{aligned} R_0(c,\psi_F^p) &= \max\{\lambda-c, p_u\lambda+(1-p_u)[1+\psi_F^p(\lambda-1)]\} \\ &= p_u\lambda+(1-p_u)[1+\psi_F^p(\lambda-1)]. \end{aligned} \tag{3.8}$$

Using (3.8), the expected payoff of a worker who invests in the hiring stage is obtained as $(1-p_q)\{1+p_u\lambda+(1-p_u)[1+\psi_F^p(\lambda-1)]\}$. Since the expected payoff from not investing remains 0, the hiring stage investment threshold is given by

$$c_{0,0}^{h*}(\psi_F^p)=(1-p_q)\{1+p_u\lambda+(1-p_u)[1+\psi_F^p(\lambda-1)]\}.$$

We have $\frac{\partial}{\partial \psi_F^p}[c_{0,0}^{h*}(\psi_F^p)] = (1-p_q)(1-p_u)(\lambda-1) > 0$, so that expected hiring stage investment is increasing in the strength of the policy. Hence, the disincentive effect of Type II promotion stage policies does not influence hiring stage investment. We have $c_{0,0}^{h*}(\psi_F^p = 1) = (1-p_q)(\lambda+1)$.

We conclude the discussion on promotion stage policies by summarizing our observations from the preceding analysis. We have:

Fact 3.2 (Impact of Promotion Stage Policies): Consider the C-C equilibrium. Then

(1): no Type I or Type II policy in the promotion stage can make all workers invest for promotion;[8]

(2): no Type I policy in the promotion stage can make all workers invest for the entry level job; and

(3): any Type I policy will stimulate greater hiring stage investment if it is introduced in the hiring stage than if it is introduced in the promotion stage.

3.4. Two Stage Policies

We now look at the effect of simultaneous policy intervention in the two stages. We consider the following cases: (1) the regulator imposes a Type I policy in both stages; and (2) he imposes a Type I policy in the hiring stage and a Type II policy in the promotion stage. The other two cases with a Type II policy in the hiring stage are omitted.

Type I in Both Stages: As mentioned before, the incentive to invest in the promotion stage is unaffected by hiring stage intervention. Hence, for all $\psi_U^h \in [0,1]$, the promotion stage investment threshold is as given in (3.4) and the ex ante expected promotion stage payoff is as given in (3.5). The hiring stage investment threshold is obtained as

$$c_{0,0}^{h*}(\psi_U^h, \psi_U^p) = [(1-p_q) + p_q\psi_U^h][2 + p_u\psi_U^p(\lambda-1)] - p_u\psi_U^h.$$

We see that the incentive to invest in the hiring stage is stronger with simultaneous intervention than intervention in any one stage: for all $\psi_U^h, \psi_F^p > 0$,

$$c_{0,0}^{h*}(\psi_U^h, \psi_U^p) - c_{0,0}^{h*}(\psi_U^h) = [(1-p_q) + p_q\psi_U^h]p_u(\lambda-1)\psi_U^p \geq 0.$$

It follows that for any $p \in [0,1]$,

$$c_{0,0}^{h*}(p = \psi_U^h, p = \psi_U^p) > c_{0,0}^{h*}(p = \psi_U^h) > c_{0,0}^{h*}(p = \psi_U^p),$$

[8] We actually have a more general result: there is no policy which can make all workers invest for promotion. We know that policy intervention in the hiring stage does not influence promotion stage investment. Hence, simultaneous intervention in the two stages cannot make all workers invest for promotion.

where the last inequality comes from (3.6). Thus, any type I policy can stimulate a greater volume of hiring stage investment if it is introduced in both stages than if it is introduced in any one stage.

Type I in Hiring Stage and Type II in Promotion Stage: In this case, the promotion stage investment threshold is as given in (3.7) and the ex ante expected promotion stage payoff is as given in (3.8). The hiring stage investment threshold is obtained as

$$c_{0,0}^{h*}(\psi_U^h, \psi_F^p) = [(1-p_q) + p_q \psi_U^h]\{1 + p_u\lambda + (1-p_u)[1 + \psi_F^p(\lambda - 1)]\} - p_u \psi_U^h.$$

We see that the incentive to invest for hiring stage investment is stronger with a Type II policy in the promotion stage than with a Type I policy in the promotion stage: for all $\psi_U^h, p \in [0,1]$,

$$c_{0,0}^{h*}(\psi_U^h, p = \psi_F^p) - c_{0,0}^{h*}(\psi_U^h, p = \psi_U^p) = [(1-p_q) + p_q \psi_U^h] p_u(\lambda - 1)(1-p) \geq 0.$$

However, the stronger incentive effect in the hiring stage may come at a cost, since Type II policies in the promotion stage have a disincentive effect on promotion stage investment.

We conclude this section by noting that two stage policies cannot make a greater fraction of workers invest in the promotion stage than promotion stage intervention alone. However, it can create better incentives for hiring stage investment, compared to policy intervention in any one stage of the game.

4. Feasible Affirmative Action in the C-C Equilibrium

This section investigates the extent to which the affirmative action policies introduced in Section 3 are feasible. We retain the assumption that the group is in the C-C equilibrium at the time of policy intervention. When the government announces an affirmative action policy, the firm calculates its ex ante expected profit under the policy. If expected profit is negative, the firm will shut down rather than operate under the policy. Hence, a *feasible* affirmative action policy is one that ensures the firm at least zero profit.

For the analysis of feasibility, it will help to calculate the expected profit of the firm in the C-C equilibrium. Recall from Section 3 that the ex ante expected promotion stage profit is $V(\pi_{0,0}^{p*}) = \pi_{0,0}^{p*}(1-p_q)(\alpha - \lambda)$. Denoting expected profit in the C-C equilibrium by $P_{0,0}^*$, we have,

$$\begin{aligned} P_{0,0}^* &= \pi_{0,0}^{h*}(1-p_q)V(\pi_{0,0}^{p*}) = \pi_{0,0}^{h*}(1-p_q) \times \pi_{0,0}^{p*}(1-p_q)(\alpha - \lambda) \\ &= (1-p_q)^3(\alpha - \lambda)(\lambda - 1) > 0. \end{aligned}$$

Since the hiring stage profit from a random worker is at most zero, positive equilibrium profit arises solely from the promotion stage contribution of qualified workers.

4.1. Hiring Stage Policies

As in the last section, we focus on Type I and Type II policies. Affirmative action in the hiring stage does not affect promotion stage investment. Hence, the only reason why hiring stage policies can reduce expected profit from $P^*_{0,0}$ is that the firm may hire workers who have not invested in the hiring stage.[9] As observed in Section 3.1, however, any policy which requires the firm to hire unclear signals with probability greater of equal to ψ_U^h makes all workers invest in the hiring stage. Hence, all Type II policies and all Type I policies with $\psi_U^h \geq \psi_U^h = \frac{2p_q-1}{2p_q-p_u}$ are feasible. Further, they ensure the firm strictly greater expected profit than $P^*_{0,0}$: for Type II policies, we have for all $\psi_F^h \in [0,1]$, $\pi^{h*}_{0,0}(\psi_F^h) = 1$. Noting that the hiring stage payoff from hiring a qualified worker is 0, the expected profit of the firm is

$$\begin{aligned} P^*_{0,0}(\psi_F^h) &= V(\pi^{p*}_{0,0}(\psi_F^h)) = (1-p_q)^2(\alpha-\lambda)(\lambda-1) \\ &> (1-p_q)^3(\alpha-\lambda)(\lambda-1) = P^*_{0,0}. \end{aligned}$$

Similarly, for Type I policies with $\psi_U^h \geq \psi_U^h$,

$$\begin{aligned} P^*_{0,0}(\psi_U^h) &= [(1-p_q)+p_q\psi_U^h]V(\pi^{p*}_{0,0}(\psi_U^h)) \\ &= [(1-p_q)+p_q\psi_U^h](1-p_q)^2(\alpha-\lambda)(\lambda-1) \\ &> P^*_{0,0}. \end{aligned}$$

We now consider Type I policies with $\psi_U^h < \psi_U^h$. The expected profit is given by

$$\begin{aligned} P^*_{0,0}(\psi_U^h) &= \pi^{h*}_{0,0}(\psi_U^h)[(1-p_q)+p_q\psi_U^h]V(\pi^{p*}_{0,0}(\psi_U^h)) - [1-\pi^{h*}_{0,0}(\psi_U^h)]p_u\psi_U^h \\ &= P^*_{0,0} + B\psi_U^h + A(\psi_U^h)^2, \end{aligned}$$

where $A = (2p_q - p_u)p_u > 0$ and $B = p_q(1-p_q)^2(\alpha-\lambda)(\lambda-1) + 2(1-p_q)p_u - p_u$.

If $B > 0$ or $p_u < \frac{p_q(1-p_q)^2(\alpha-\lambda)(\lambda-1)}{2p_q-1}$, we have

$$\left.\frac{\partial P^*_{0,0}(\psi_U^h)}{\partial \psi_U^h}\right|_{\psi_U^h < \tilde{\psi}_U^h} = B + 2A\psi_U^h > 0.$$

Hence, all Type I and Type II policies yield a strictly greater profit than $P^*_{0,0}$.[10]

If $B < 0$ and both roots of $P^*_{0,0} + B\psi_U^h + A(\psi_U^h)^2 = 0$ are real and positive, then there is a range of Type I hiring stage policies which are not feasible. However, the optimal

[9] Recall that hiring an unqualified worker leads to a payoff of -1 for the firm.

[10] In this case, the probability of an unqualified worker emitting an unclear signal is low. Hence, the possibility of the firm to hiring an unqualified worker under a Type I policy is low.

hiring stage policy cannot be a Type I policy with $\psi_U^h < \psi_U^h$: $P_{0,0}^*(\psi_U^h)$ is convex on the interval $[0, \psi_U^h]$, since

$$\left. \frac{\partial^2 P_{0,0}^*(\psi_U^h)}{\partial(\psi_U^h)^2} \right|_{\psi_U^h < \tilde{\psi}_U^h} = 2A > 0.$$

Hence, for all $\psi_U^h \in [0, \psi_U^h]$, $P_{0,0}^*(\psi_U^h) \leq \min\limits_{\psi_U^h \in [\tilde{\psi}_U^h, 1]} P_{0,0}^*(\psi_U^h) = P_{0,0}^*(\psi_U^h)$.

To conclude the analysis of feasible hiring stage policies, we summarize our observations in the result below:

Proposition 4.1 (Feasible Hiring Stage Policies): Consider the C-C equilibrium. Then

(1): all Type II policies and Type I policies with $\psi_U^h \geq \psi_U^h = \frac{2p_q - 1}{2p_q - p_u}$ yield strictly greater expected profit than $P_{0,0}^*$; and

(2): if $p_u < \frac{p_q(1-p_q)^2(\alpha-\lambda)(\lambda-1)}{2p_q-1}$, then all Type I policies yield strictly greater expected profit than $P_{0,0}^*$.

4.2. Feasible Promotion Stage Policies

We start by noting that at $\psi_F^p = 1$, no worker invests in the promotion stage. Yet the firm is required to promote every worker, regardless of test score. Clearly, this policy should lead to an expected loss for the firm. Hence, we first look at the effect of Type II policies on expected profit. The ex ante expected promotion stage profit under type II policies is given by

$$\begin{aligned} V(\pi_{0,0}^{p*}(\psi_F^p)) &= \pi_{0,0}^{p*}(\psi_F^p)(\alpha - \lambda) - [1 - \pi_{0,0}^{p*}(\psi_F^p)][p_u + (1 - p_u)\psi_F^p]\lambda \\ &= \pi_{0,0}^{p*}(\psi_F^p)(\alpha - \lambda) + \pi_{0,0}^{p*}(\psi_F^p)[p_u + (1 - p_u)\psi_F^p]\lambda \\ &\quad -[p_u + (1 - p_u)\psi_F^p]\lambda. \end{aligned}$$

Hence, total expected profit under Type II policies is

$$\begin{aligned} P_{0,0}^*(\psi_F^p) &= \pi_{0,0}^{h*}(\psi_F^p)(1 - p_q)V(\pi_{0,0}^{p*}(\psi_F^p)) \\ &= c_0^{p*}(\psi_F^p)(1 - p_q)(\alpha - \lambda) + c_0^{p*}(\psi_F^p)(1 - p_q)[p_u + (1 - p_u)\psi_F^p]\lambda, \end{aligned}$$

where $c_0^{p*}(\psi_F^p)$ denotes the promotion stage investment threshold under a type II policy. The expected profit function $P_{0,0}^*(\psi_F^p)$ is characterized in the following lemma:

Lemma 4.1 (Expected Profit under Type II Policies): Consider the C-C equilibrium. Then for $\psi_F^p \in [0, 1]$, we have

$$P_{0,0}^*(\psi_F^p) = E - D\psi_F^p - C(\psi_F^p)^2, \tag{4.1}$$

where the coefficients are defined as

$$\begin{aligned}
C &= (1-p_q)(\lambda-1)(1-p_u)^2 p_u\lambda + (1-p_q)^2(\lambda-1)(1-p_u)^2\lambda, \\
D &= (1-p_q)(\alpha-\lambda)(\lambda-1)(1-p_u) - (1-p_q)(\lambda-1)(1-p_u)^2 p_u\lambda \\
&\quad +(1-p_q)(\lambda-1)(1-p_u)p_u\lambda + (1-p_q)^2[2+p_u(\lambda-1)](1-p_u)\lambda \\
&\quad +(1-p_q)^2(1-p_u)(\lambda-1)p_u\lambda, \\
E &= (1-p_q)(\alpha-\lambda)(\lambda-1)(1-p_u) + (1-p_q)(\lambda-1)(1-p_u)p_u\lambda \\
&\quad -(1-p_q)^2[2+p_u(\lambda-1)]p_u\lambda;
\end{aligned}$$

and C, D and $E > 0$.

Proof: In the Appendix. ■

Lemma 4.1 implies that there is a class of Type II promotion stage policies which are not feasible. We have:

Proposition 4.2 (Feasible Type II Policies): Consider the C-C equilibrium. Then there is $\psi_F^p \in (0,1)$ such that $P_{0,0}^*(\psi_F^p) \geq 0$ if and only if $\psi_F^p \leq \frac{1}{2C}(D + \sqrt{D^2 + 4CE})$.

Proof: From (4.1), we have $P_{0,0}^*(\psi_F^p = 0) = E > 0$. However, $P_{0,0}^*(\psi_F^p = 1) = -\pi_{0,0}^{h*}(\psi_F^p = 1)(1-p_q)\lambda = -(1-p_q)^2\lambda(\lambda+1) < 0$. Since $C, D > 0$, $P_{0,0}^*(\psi_F^p)$ is strictly decreasing on $[0,1]$. Hence, there is $\psi_F^p \in (0,1)$ such that $P_{0,0}^*(\psi_F^p = \psi_F^p) = 0$. To complete the proof, note that ψ_F^p is obtained as the unique real root of $E - D\psi_F^p - C(\psi_F^p)^2 = 0$. Hence, $\psi_F^p = \frac{1}{2C}(D + \sqrt{D^2 + 4CE})$. ■

It now remains to consider the effect of Type I policies on expected profit. In this case, the ex ante expected promotion stage profit is given by

$$\begin{aligned}
V(\pi_{0,0}^{p*}(\psi_U^p)) &= \pi_{0,0}^{p*}(\psi_U^p)[(1-p_q) + p_q\psi_U^p](\alpha-\lambda) - [1 - \pi_{0,0}^{p*}(\psi_U^p)]p_u\psi_U^p\lambda \\
&= \pi_{0,0}^{p*}(\psi_U^p)(1-p_q)(\alpha-\lambda) + \pi_{0,0}^{p*}(\psi_U^p)p_q(\alpha-\lambda)\psi_U^p \\
&\quad +\pi_{0,0}^{p*}(\psi_U^p)p_u\lambda\psi_U^p - p_u\lambda\psi_U^p.
\end{aligned}$$

Hence, total expected profit under promotion stage Type I policies is

$$\begin{aligned}
P_{0,0}^*(\psi_U^p) &= \pi_{0,0}^{h*}(\psi_U^p)(1-p_q)V(\pi_{0,0}^{p*}(\psi_U^p)) \\
&= c_0^{p*}(\psi_U^p)(1-p_q)^2(\alpha-\lambda) + c_0^{p*}(\psi_U^p)p_q(1-p_q)(\alpha-\lambda)\psi_U^p \\
&\quad +c_0^{p*}(\psi_U^p)(1-p_q)p_u\lambda\psi_U^p - c_{0,0}^{h*}(\psi_U^p)(1-p_q)p_u\lambda\psi_U^p.
\end{aligned}$$

The expected profit function $P_{0,0}^*(\psi_U^p)$ is characterized in the following lemma:

Lemma 4.2 (Expected Profit under Type I Policies): Consider the C-C equilibrium. Then for $\psi_U^p \in [0,1]$, we have

$$P_{0,0}^*(\psi_U^p) = P_{0,0}^* + G\psi_U^p + F(\psi_U^p)^2,$$

where the coefficients are defined as

$$\begin{aligned} F &= (1-p_q)(\lambda-1)\left[p_q(\alpha-\lambda)(p_q-p_u)+p_u\lambda(p_q-p_u)-(1-p_q)p_u^2\lambda\right] \\ G &= (1-p_q)^2[(\alpha-\lambda)(\lambda-1)(p_q-p_u)+p_q(\alpha-\lambda)(\lambda-1)+(\lambda-1)p_u\lambda-2p_u\lambda] \end{aligned}$$

and $F, G > 0$ if

$$\alpha-\lambda > \frac{[2+p_u(\lambda-1)-(\lambda-1)(1-p_u)]p_u\lambda}{(\lambda-1)(p_q-p_u)} \tag{4.2}$$

Proof: In the Appendix. ■

If F and G are positive, $P^*_{0,0}(\psi^p_U)$ is strictly increasing on $[0,1]$. This allows us to provide a simple interpretation of (4.2): Recall that $\alpha-\lambda$ is the net profit from promoting a worker who has invested in the promotion stage. Note also that Type I policies increase promotion stage investment, so that the firm is essentially trading off the expected gain from promoting a greater fraction of qualified workers with the expected loss from promoting workers who have not invested in the promotion stage. Condition (4.2) states that if the net profit from promoting a qualified worker is sufficiently high, then the gain from affirmative action is greater than the loss, whereby firm profit increases in the strength of the Type I policy introduced. We have

Proposition 4.3 (Feasible Type I Policies): Consider the $C-C$ equilibrium. If Condition (4.2) holds, all Type I policies are feasible and yield strictly greater profit than $P^*_{0,0}$.

To conclude, we note that if Condition (4.2) does not hold, there is a class of Type I policies which are not feasible, provided the profit function is convex and both roots of $P^*_{0,0}+G\psi^p_U+F(\psi^p_U)^2=0$ are real and positive.

5. The Optimal Affirmative Action Policy

We have seen that policy intervention can have a positive impact on expected educational investment in both stages of the game. In this section, we characterize the policy that will induce the best non-discriminatory equilibrium. The optimal affirmative action policy is denoted as the vector $(\bar{\psi}^h, \bar{\psi}^p)$.

We start with the characterization of the optimal hiring stage policy. Recall that any equilibrium firm strategy will hire and promote pass signals with probability one. Hence, we can focus on Type I and Type II policies introduced in section 3. We have:

Lemma 5.1 (Optimal Hiring Stage Policy): $\bar{\psi}^h$ requires the firm to hire all workers with unclear signals.

Proof: To prove the result, we proceed in a number of steps:

Step 1. The equilibrium promotion stage action of a worker does not depend on the equilibrium hiring stage action of the firm.

We do not prove this formally as the underlying rationale is simple. Any worker facing firm strategy (ψ^h, ψ^p) either invests in the hiring stage or does not. In case of the latter, he is not eligible for promotion and a change in ψ^h will not make him eligible. Hence, his equilibrium promotion stage action need not change in response to a new hiring policy of the firm. In case of the former, the worker is eligible for the promotion stage and his ex ante expected promotion stage payoff is $R(c, \psi^p)$, where

$$R(c, \psi^p) = \begin{cases} \max\{[(1-p_q) + p_q \psi_U^p]\lambda - c; 1 + p_u \psi_U^p(\lambda - 1)\} & \text{if } \psi^p \text{ is Type I} \\ \\ \max\{\lambda - c; p_u\lambda + (1-p_u)[1 + \psi_F^P(\lambda - 1)]\} & \text{if } \psi^p \text{ is Type II} \end{cases}$$

Again, a change in hiring policy will not affect this payoff, so that his promotion stage best response will not change.

Step 2. $\bar{\psi}^h$ leads to all workers investing in the hiring stage.[11]

Proof: Consider any ψ^p. By Step 1, the ex ante expected promotion stage payoff of a worker is $R(c, \psi^p)$. Consider $\bar{\psi}^h$ and a worker with $c = 1$. If he invests in the hiring stage, his total expected payoff is $[1 + R(c, \psi^p)] - 1$. If he does not invest, his total expected payoff is p_u. We have $[1 + \min R(c, \psi^p)] - 1 = [1 + R_0(c)] - 1 = 1 > p_u$, where the minimum is taken over the promotion stage policy space. Since investment is optimal for the highest cost type, all workers will invest under $\bar{\psi}^h$.

Step 3: $P^*(\bar{\psi}^h, \psi^p) > P^*(\psi^h, \psi^p)$ for all ψ^h and ψ^P.

Proof: By Step 2, $\pi^{h*}(\bar{\psi}^h, \psi^p) = 1$ for any ψ^p. Hence, expected profit under $\bar{\psi}^h$ is $P^*(\bar{\psi}^h, \psi^p) = V(\pi^{p*}(\bar{\psi}^h, \psi^p))$. For any policy with $\psi_U^h < 1$, we have $P^*(\psi^h, \psi^p) = [(1-p_q) + p_q \psi_U^h] V(\pi^{p*}(\psi_U^h, \psi^p)) < P^*(\bar{\psi}^h, \psi^p)$.

To complete the proof of Lemma 5.1, it remains to be shown that workers earn at least as much expected payoff under $\bar{\psi}^h$ as under any other policy. Denoting the equilibrium expected payoff of a worker by $U^*(.,.)$, we have

Step 4. $U^*(\bar{\psi}^h, \psi^p) \geq U^*(\psi^h, \psi^p)$ for all ψ^h and ψ^p.

We classify workers into the following types: (i) those who invest in the hiring stage and draw the pass signal; (ii) those who invest in the hiring stage and draw the unclear signal; (iii) those who do not invest in the hiring stage and draw the unclear signal; and

[11] If all workers invest under $\bar{\psi}^H$, the only hiring stage signals observed are pass and unclear. Pass signals are hired for sure in any equilibrium. By hiring unclear signals for sure, the firm hires the entire population.

(iv) those who do not invest in the hiring stage and draw the fail signal. The following table compares the expected payoff of each type under $\bar{\psi}^h$ with that under alternative policies.

Worker Type	$U^*(\psi^h, \psi^p)$ if ψ^h is Type I	$U^*(\psi^h, \psi^p)$: if ψ^h is Type II	$U^*(\bar{\psi}^h, \psi^p)$
(i)	$1 + R(c, \psi^p)$	$1 + R(c, \psi^p)$	$1 + R(c, \psi^p)$
(ii)	$\psi_U^h[1 + R(c, \psi^p)]$	$1 + R(c, \psi^p)$	$1 + R(c, \psi^p)$
(iii)	ψ_U^h	1	1
(iv)	0	ψ_F^h	1

Noting that $\psi_U^h, \psi_F^h \in [0, 1]$, the result follows by inspection. ■

We now consider the optimal promotion stage policy. Clearly, the firm will prefer workers to invest in education. If educational investment was observable and verifiable, the optimal promotion policy is to promote a worker if and only if he invests and keep him at the entry level job if he does not. We, therefore, start by asking if all workers can be made to invest in the promotion stage.

Recall from Section 3 that the highest expected promotion stage investment under any policy is $(1 - p_u)(\lambda - 1)$, and this is achieved by policies with $\psi_U^p = 1$. For any p_u, $(1 - p_u)(\lambda - 1) \geq 1$ if and only if $\lambda \geq 1 + \frac{1}{1-p_u}$.[12] In this case, therefore, the optimal promotion stage policy is to require the firm to promote all unclear signals. We state this simple case as the following Lemma:

Lemma 5.2 (Optimal Affirmative Action - I): Let $\lambda \geq 1 + \frac{1}{1-p_u}$. Then $(\bar{\psi}^h, \bar{\psi}^p)$ requires the firm to hire and promote all unclear signals.

A more interesting situation arises when $\lambda < 1 + \frac{1}{1-p_u}$. Since expected promotion stage investment increases in the probability with which the firm promotes unclear signals and decreases in the probability with which it promotes fail signals, if a policy with $\psi_U^p = 1$ does not make all workers invest in the promotion stage, then there is no policy which achieves this.

We first ask if the firm can gain by promoting fail signals with positive probability. Recall that the cost of promotion stage affirmative action derives from the firm having to promote unqualified workers. Since policies with $\psi_F^p > 0$ reduce promotion stage investment relative to policies with $\psi_F^p = 0$, the former cannot lead to greater profit for the firm. In other words, no Type II promotion policy with $\psi_F^p > 0$ can be optimal. We formalize this argument in the following result:

[12] That is, if and only if wages in the promotion level job are sufficiently high, will all workers invest in the promotion stage.

Lemma 5.3 (Suboptimality of Type II Policies): Let $\lambda < 1 + \frac{1}{1-p_u}$. Then

$$V(\pi^{p*}(\bar{\psi}^h, \psi_F^p > 0)) < V(\pi^{p*}(\bar{\psi}^h, \psi_F^p = 0)).$$

Proof: Given $\bar{\psi}^h$, conditional expected profit function under a Type II policy is

$$\begin{aligned} V(\pi^{p*}(\bar{\psi}^h, \psi_F^p)) &= \pi^{p*}(\bar{\psi}^h, \psi_F^p)(\alpha - \lambda) - [1 - \pi^{p*}(\bar{\psi}^h, \psi_F^p)][p_u + (1-p_u)\psi_F^p]\lambda \\ &= (1-p_u)(\lambda - 1)(1 - \psi_F^p)[(\alpha - \lambda) + p_u\lambda + (1-p_u)\lambda\psi_F^p] \\ &\quad -[p_u + (1-p_u)\psi_F^p]\lambda. \text{ [Using (3.7)]} \end{aligned}$$

Differentiating with respect to ψ_F^p, we have

$$\begin{aligned} \frac{\partial V(\pi^{p*}(\bar{\psi}^h, \psi_F^p))}{\partial \psi_F^p} &= (1-p_u)^2(\lambda - 1)\lambda - (1-p_u)^2(\lambda-1)\lambda\psi_F^p \\ &\quad -(1-p_u)(\lambda-1)[(\alpha-\lambda) + p_u\lambda + (1-p_u)\lambda\psi_F^p] - (1-p_u)\lambda \\ &= -(1-p_u)^2(\lambda-1)\lambda\psi_F^p - (1-p_u)\lambda \\ &\quad -(1-p_u)(\lambda-1)[(\alpha-\lambda) + (1-p_u)\lambda\psi_F^p] - (1-p_u)(\lambda-1)\lambda \\ &< 0. \end{aligned}$$

Hence, the result follows. ■

We now look at Type I policies which promote unclear signals with probability $\psi_U^p > 0$. Such policies have two conflicting effects: they increase expected profit relative to $\psi_U^p = 0$ since the unclear signals being promoted include workers who have invested in the promotion stage. At the same time, they decrease expected profit, since unqualified workers are also promoted with positive probability. Depending on which effect dominates, the expected promotion stage profit may be increasing or decreasing in ψ_U^p. This is the subject of the following result:

Lemma 5.4 (Convexity of Conditional Profit Function): Let $\lambda < 1 + \frac{1}{1-p_u}$. Then

(1): $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p < 1)) < V(\pi^{p*}(\bar{\Delta}^H, \psi_U^p = 1))$ if and only if

$$\alpha - \lambda > \frac{[1 - (1-p_u)(\lambda-1)]p_u\lambda}{(\lambda-1)[(1-p_u) - (1-p_q)^2]}$$

(2): $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p = 0)) > V(\pi^{p*}(\bar{\psi}^h, \psi_U^p > 0))$ if and only if

$$\alpha - \lambda < \frac{[1 - (1-p_u)(\lambda-1)]p_u\lambda}{(\lambda-1)[(1-p_u) - (1-p_q)^2]}.$$

Proof: Given $\bar{\psi}^h$, conditional expected profit function under a Type I policy is

$$\begin{aligned} V(\pi^{p*}(\bar{\psi}^h, \psi_U^p)) &= \pi^{p*}(\bar{\psi}^h, \psi_U^p)[(1-p_q) + p_q\psi_U^p](\alpha - \lambda) \\ &\quad -[1-\pi^{p*}(\bar{\psi}^h, \psi_U^p)]p_u\psi_U^p\lambda \\ &= [(1-p_q)(\lambda-1) + (p_q - p_u)(\lambda-1)\psi_U^p] \\ &\quad \times[(1-p_q)(\alpha-\lambda) + p_q(\alpha-\lambda)\psi_U^p + p_u\lambda\psi_U^p] - p_u\lambda\psi_U^p \end{aligned}$$

Differentiating with respect to p_{Δ^P}, we have

$$\begin{aligned} \frac{\partial V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))}{\partial \psi_U^p} &= (1-p_q)(\alpha-\lambda)(\lambda-1)(p_q-p_u) \\ &\quad +p_q(1-p_q)(\alpha-\lambda) + (1-p_q)(\lambda-1)p_u\lambda \\ &\quad -p_u\lambda + 2\psi_U^p[p_q(\alpha-\lambda)(\lambda-1)(p_q-p_u) + (\lambda-1)(p_q-p_u)p_u\lambda] \end{aligned}$$

and

$$\frac{\partial^2 V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))}{\partial (\psi_U^p)^2} = 2[p_q(\alpha-\lambda)(\lambda-1)(p_q-p_u) + (\lambda-1)(p_q-p_u)p_u\lambda] > 0,$$

so that $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ is convex. Now,

$$\begin{aligned} V(\pi^{p*}(\bar{\psi}^h, \psi_U^p &= 1)) = (1-p_u)(\lambda-1)[(\alpha-\lambda) + p_u\lambda] - p_u\lambda \\ &\gtrless (1-p_q)^2(\alpha-\lambda)(\lambda-1) = V(\pi^{p*}(\bar{\psi}^h, \psi_U^p = 0)) \end{aligned}$$

if and only if

$$(1-p_u)(\lambda-1)(\alpha-\lambda) - p_u\lambda[1 + (1-p_u)(\lambda-1)] \gtrless (1-p_q)^2(\alpha-\lambda)(\lambda-1)$$

$$\text{or,} \qquad \alpha - \lambda \gtrless \frac{1-(1-p_u)(\lambda-1)}{(\lambda-1)[(1-p_u)-(1-p_q)^2]}p_u\lambda. \tag{5.1}$$

If (5.1) holds with $>$ sign, $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ is increasing and convex, whereby (5.4.1) is established. If, however, (5.1) holds with $<$ sign, $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ is decreasing and convex. Then $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p = 0)) > V(\pi^{p*}(\bar{\psi}^h, \psi_U^p > 0))$. By Lemma 5.3, the second statement follows. ■

Condition (5.1), which determines, the slope of $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ has a simple interpretation: if the net profit from promoting a qualified worker is sufficiently high, then the expected gain from promoting qualified workers with unclear signals dominates the expected loss from promoting unqualified workers with unclear signals. If, however, net profit is low, then the reverse is true. Note that the convexity of the conditional profit function $V(\pi^{p*}(\bar{\psi}^h, \psi^p_U))$ ensures that we have a corner solution, whereby the firm does best either by promoting all unclear signals or by promoting none of them. We now come to the main result of this section:

Proposition 5.5 (Optimal Affirmative Action - II): Let $\lambda < 1 + \frac{1}{1-p_u}$. Then

(1): $(\bar{\psi}^h, \bar{\psi}^p)$ requires the firm to hire and promote all unclear signals if

$$\alpha - \lambda > \frac{1-(1-p_u)(\lambda-1)}{(\lambda-1)[(1-p_u)-(1-p_q)^2]} p_u \lambda$$

(2): $(\bar{\psi}^h, \bar{\psi}^p)$ requires the firm to hire all unclear signals and promote only pass signals if

$$\alpha - \lambda < \frac{1-(1-p_u)(\lambda-1)}{(\lambda-1)[(1-p_u)-(1-p_q)^2]} p_u \lambda.$$

Lastly, we come to the question of feasibility. Note that the optimal affirmative action policy $(\bar{\psi}^h, \bar{\psi}^p)$ is selecting either the $L-L$ equilibrium or the $L-C$ equilibrium, depending on whether the conditional expected profit function $V(\pi^{p*}(\bar{\psi}^h, \psi^p_U))$ is increasing or decreasing in ψ^p_U. Since these equilibria exist, they must be feasible. Hence, the optimal affirmative action policy $(\bar{\psi}^h, \bar{\psi}^p)$ is feasible. We highlight this as the last result in the paper. While the underlying rationale is clear, we include a technical proof in the Appendix.

Proposition 5.6 (Feasibility): $(\bar{\psi}^h, \bar{\psi}^p)$ is feasible.

Proof. In the Appendix. ■

In the next section, we discuss the implications of our results; the assumptions underlying them; and possible extensions of our research.

6. Discussion and Extensions

As observed in the last section, the optimal policy $(\bar{\psi}^h, \bar{\psi}^p)$ locates the economy either in the L-C or in the L-L equilibrium. Hence, it is selecting among the four equilibria introduced in Fryer (2006) [10]. This is due to the assumption that the cost of educational investment c is distributed uniformly. Uniformity is sufficient to ensure that conditional expected promotion stage profit $V(\pi^{p*}(\bar{\psi}^h, \psi^p_U))$ is convex in ψ^p_U, whereby there is a corner solution to the profit maximization problem. However, uniformity is not necessary for a corner solution and any weakly convex distribution function for c will give the same result. With a general continuous distribution, however, we may have an interior solution and $(\bar{\psi}^h, \bar{\psi}^p)$ may require the firm to promote unclear signals with $\psi^p_U \in (0, 1)$.

The optimal hiring stage policy $\bar{\psi}^h$ requires the firm to hire all unclear signals. As mentioned before, this leads to the entire population being hired. One could, of course, think of alternative specifications where the optimal policy will not lead to the entire population being hired. However, it is safe to conjecture that even in this case, the optimal policy will provide more incentive to invest in the early stage of a worker's career than in the later stage.

Since all workers invest in the hiring stage at the optimum, the entire population is eligible for promotion. Hence, conditional on $\bar{\psi}^h$, the promotion stage in this model is similar to the job assignment problem in Coate and Loury (1993) [8], if we allow general distributions for the promotion stage signals of qualified and unqualified workers.[13] This may raise a question as to why we need the hiring stage in the model. To address this, we emphasize that all workers invest in the hiring stage if only if the probability with which the firm hires unclear signals exceeds a certain threshold.[14] Hence, there is a definite possibility that the economy can be stuck in a bad hiring stage outcome. Further, the optimal hiring stage policy ensures that all workers earn positive payoff even if they are not promoted. This creates a stronger incentive to invest for the more lucrative job as compared to single stage models such as Coate and Loury (1993) [8]: in this model, a worker who is not promoted gets 1 if he did not invest for promotion and $1 - c \geq 0$ if he did. In [8], however, a worker who fails to be assigned to the more lucrative job gets 0 if he did not invest and $-c < 0$ if he did.

One may also ask to what extent has the discrete signal space in our model influenced our results? Recall that the key feature of statistical discrimination models which emerged following Coate and Loury (1993) [8] is the possibility of multiple equilibria under affirmative action. Our assumption of a discrete signal space does not rule out this possibility.

Note that affirmative action impacts the standard used by a firm to assign workers to the more lucrative job. Multiple equilibria arise in [8] because a change in the assignment standard has two potentially conflicting effects: as the assignment standard falls, workers need a lower test score to be assigned to the more lucrative job. Hence, the *rationing effect* unambiguously increases the expected fraction of workers who gain this assignment. On the other hand, a fall in the assignment standard may either stimulate or reduce educational investment. Hence, the *incentive effect* may increase or decrease the expected fraction of workers assigned to the more lucrative job. The net effect of a change in the assignment standard is thus ambiguous.

The same ambiguity is present in our model: consider Type II policies which require

[13] Being promoted corresponds to being assigned to the more lucrative job in [8].

[14] For any promotion policy ψ^P and any hiring policy with $\psi_U^H \in [0,1]$, the hiring stage investment threshold is $c^{H*}(\psi_U^H, \psi^P) = [(1-p_q) + p_q\psi_U^H][1 + R(c,\psi^P)] - p_u\psi_U^H$. This equals 1 for $\psi_U^H \geq \frac{1-(1-p_q)[1+R(c,\psi^P)]}{p_q[1+R(c,\psi^P)]-p_u}$.

the firm to promote unclear signals for sure and fail signals with probability $\psi_F^p \in [0,1]$. As ψ_F^p rises, workers find it easier to get promoted. At the same time, the expected fraction of workers investing in the promotion stage falls. The net effect on the expected fraction of workers promoted is ambiguous, whereby there is a possibility of multiple equilibria.[15] Nevertheless, an extension of our analysis to general continuous signal distributions is an immediate agenda for research.

It should be mentioned that the optimal policy in our model is *color blind* in the sense that the group identity of a worker plays no role in his treatment by the firm. Recall that we have assumed groups to be ex ante identical, so that discrimination is driven by negative prior beliefs on the part of the firm. If groups are not ex ante identical and persistent disadvantages make members of one group less likely to attain higher test scores, then color blind intervention may be inefficient and the optimal policy may be *color sighted* (see Benoit (1999) [3] and Fryer and Loury (2005b) [12]).[16].

It should also be pointed out that our model may be reinterpreted as a model of affirmative action in education and employment, where the hiring stage corresponds to college admission and the promotion stage to the job market. If we assume that the college cares about the quality of incoming students and both college and firm have common prior on whether an applicant from a particular group is qualified for college, then the present model qualifies with virtually no significant modification being needed.[17] It is interesting to note that in this case, our model suggests that universal education is a desirable outcome.

Lastly, we have assumed fixed wages in both stages of the game. As pointed out by Moro and Norman (2003, 2004) [15], [14], affirmative action changes the profitability of hiring different groups of workers for all firms, so that equilibrium wage levels should change in principle. If we consider a general equilibrium environment with variable wages, then affirmative action policies increase educational investment unambiguously, even though the beneficiary group may be worse off due to increasing income inequality. An important agenda for future research is to extend our analysis to a general equilibrium model.

7. Conclusion

This paper introduced affirmative action policies in a model where workers from a particular group face statistical discrimination in both hiring and promotion decisions in the labor market. We first considered the most discriminatory equilibrium and showed

[15] A formal demonstration is available on request.

[16] See also Chan and Eyster (2003) [5] on the inefficiency of color blind policies in college admissions and Fryer *et. al.* (2006) [9] for a related argument in the same context.

[17] This is the route taken by Fang and Fryer [13], though their model incorporates neighborhood effects and focuses on the optimal timing of affirmative action over the life cycle of a worker.

that moderate affirmative action can stimulate educational investment in the discriminated group. Further, the disincentive effect of affirmative action, whereby such policies actually reduce educational investment in the discriminated group, is relevant only for policies which require the firm to promote workers who are clearly unqualified.

We then characterized the optimal affirmative action plan in our model. The optimal hiring stage policy requires the firm to hire all workers with unclear signals. This, in effect, leads to all workers being hired. However, the optimal promotion stage policy either requires the firm to promote all unclear signals or to promote pass signals exclusively, depending on the net profit from promoting a qualified worker. Hence, our model suggests that while it is always beneficial to intervene in the hiring stage, the decision to intervene in the promotion stage requires caution. Lastly, the optimal policy allows the firm positive profit. Hence, concerns about affirmative action being too costly to implement for the firm prove to be unfounded in our model.

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8. Appendix

This section contains proofs which were omitted from the body of the paper. We first consider the characterization of expected profit under Type II policies in the promotion stage.

Proof of Lemma 4.1: In this case, total expected profit is given by

$$\begin{aligned} P^*_{0,0}(\psi^p_F) &= \pi^{h*}_{0,0}(\psi^p_F)(1-p_q)V(\pi^{p*}_{0,0}(\psi^p_F)) \\ &= c^{p*}_0(\psi^p_F)(1-p_q)(\alpha-\lambda) + c^{p*}_0(\psi^p_F)(1-p_q)[p_u+(1-p_u)\psi^p_F]\lambda, \end{aligned}$$

Expanding the first term in the above expression, we have

$$\begin{aligned} & (1-p_q)(\alpha-\lambda)\times(1-p_u)(\lambda-1)(1-\psi^p_F) \\ = \quad & (1-p_q)(\alpha-\lambda)(\lambda-1)(1-p_u) - (1-p_q)(\alpha-\lambda)(\lambda-1)(1-p_u)\psi^p_F. \end{aligned}$$

Similarly, the second term can be expressed as

$$\begin{aligned}
& (1-p_q)[p_u+(1-p_u)\psi_F^p]\lambda \times (1-p_u)(\lambda-1)(1-\psi_F^p) \\
= \; & (1-p_q)[p_u+(1-p_u)\psi_F^p]\lambda(1-p_u)(\lambda-1) \\
& -(1-p_q)[p_u+(1-p_u)\psi_F^p]\lambda(1-p_u)(\lambda-1)\psi_F^p \\
= \; & (1-p_q)(\lambda-1)(1-p_u)p_u\lambda + (1-p_q)(\lambda-1)(1-p_u)^2 p_u\lambda\psi_F^p \\
& -(1-p_q)(\lambda-1)(1-p_u)p_u\lambda\psi_F^p - (1-p_q)(\lambda-1)(1-p_u)^2 p_u\lambda(\psi_F^p)^2,
\end{aligned}$$

and the third term as

$$\begin{aligned}
& (1-p_q)[p_u+(1-p_u)\psi_F^p]\lambda \times (1-p_q)[1+p_u\lambda+(1-p_u)\{1+\psi_F^p(\lambda-1)\}] \\
= \; & [p_u+(1-p_u)\psi_F^p]\lambda(1-p_q)^2[\{2+p_u(\lambda-1)\}+(1-p_u)(\lambda-1)\psi_F^p] \\
= \; & (1-p_q)^2[2+p_u(\lambda-1)]p_u\lambda + (1-p_q)^2[2+p_u(\lambda-1)](1-p_u)\lambda\psi_F^p \\
= \; & +(1-p_q)^2(1-p_u)(\lambda-1)p_u\lambda\psi_F^p + (1-p_q)^2(1-p_u)^2(\lambda-1)\lambda(\psi_F^p)^2.
\end{aligned}$$

Collecting the coefficients of ψ_F^p and $(\psi_F^p)^2$ yields the desired expression.

It is clear by inspection that the coefficients of $(\psi_F^p)^2$ and ψ_F^p, that is, C and D, are greater than zero. To prove that $E > 0$, we write the expression for E as

$$E = (1-p_q)\left[(\lambda-1)(1-p_u)\{(\alpha-\lambda)+p_u\lambda\} - (1-p_q)\{2+p_u(\lambda-1)]p_u\lambda\}\right]$$

Clearly, $E > 0$ if and only if

$$(1-p_u)(\lambda-1)[(\alpha-\lambda)+p_u\lambda] > (1-p_q)[2+p_u(\lambda-1)]p_u\lambda$$

$$\text{or, } \frac{(1-p_u)(\lambda-1)}{(1-p_q)[2+p_u(\lambda-1)]} > \frac{p_u\lambda}{(\alpha-\lambda)+p_u\lambda} = \frac{1}{1+(\frac{\alpha-\lambda}{p_u\lambda})}.$$

Recall from Section 2 that $\frac{(1-p_u)(\lambda-1)}{(1-p_q)[2+p_u(\lambda-1)]} = \pi_{0,1}^{p*}$, where $\pi_{0,1}^{p*}$ is the promotion stage belief in the $C-L$ equilibrium. Hence,

$$\frac{(1-p_u)(\lambda-1)}{(1-p_q)[2+p_u(\lambda-1)]} > \hat{\pi}_{0,1}^p = \frac{1}{1+(\frac{\alpha-\lambda}{p_u\lambda})p_q}$$

Since $\frac{1}{1+(\frac{\alpha-\lambda}{p_u\lambda})p_q} > \frac{1}{1+(\frac{\alpha-\lambda}{p_u\lambda})}$, we have $E > 0$. ■

We now characterize expected profit under Type I policies in the promotion stage.

Proof of Lemma 4.2: The first part of the result follows from expanding the terms of the profit function $P^*_{0,0}(\psi^p_F)$ and collecting the coefficients of ψ^p_F and $(\psi^p_F)^2$. To prove the second part, write (4.2) as

$$\begin{aligned}(\alpha-\lambda)(\lambda-1)(p_q-p_u) &> [2+p_u(\lambda-1)-(\lambda-1)(1-p_u)]p_u\lambda \\ &= [2-(\lambda-1)(1-2p_u)]p_u\lambda \\ &= 2p_u\lambda-(\lambda-1)p_u\lambda+2(\lambda-1)p_u^2\lambda.\end{aligned}$$

$$\text{or, } (\alpha-\lambda)(\lambda-1)(p_q-p_u)+(\lambda-1)p_u\lambda > 2p_u\lambda.$$

This implies $(\alpha-\lambda)(\lambda-1)(p_q-p_u)+p_q(\alpha-\lambda)(\lambda-1)+(\lambda-1)p_u\lambda > 2p_u\lambda$, since $p_q(\alpha-\lambda)(\lambda-1) > 0$. Hence, $G > 0$.

To prove that $F > 0$, write the expression for F as

$$\begin{aligned}F &= (1-p_q)(\lambda-1)\left[p_q(\alpha-\lambda)(p_q-p_u)+p_up_q\lambda-p_u^2\lambda-(1-p_q)p_u^2\lambda\right] \\ &= (1-p_q)(\lambda-1)[p_q(\alpha-\lambda)(p_q-p_u)+p_up_q\lambda(1+p_u)-2p_u^2\lambda].\end{aligned}$$

$F > 0$ if and only if

$$p_q(\alpha-\lambda)(p_q-p_u)+p_up_q\lambda(1+p_u) > 2p_u^2\lambda. \qquad (8.1)$$

Note that (4.2) can be written as

$$\begin{aligned}&(\alpha-\lambda)(\lambda-1)(p_q-p_u)+(\lambda-1)(1-2p_u)p_u\lambda > 2p_u\lambda \\ &\text{or, } p_q(\alpha-\lambda)(\lambda-1)(p_q-p_u)+p_q(\lambda-1)(1-2p_u)p_u\lambda > 2p_up_q\lambda > 2p_u^2\lambda \\ &\text{or, } [p_q(\alpha-\lambda)(p_q-p_u)+p_q(1-2p_u)p_u\lambda](1-p_q)(\lambda-1) > 2p_u^2\lambda(1-p_q) \\ &\text{or, } [p_q(\alpha-\lambda)(p_q-p_u)+p_q(1-2p_u)p_u\lambda] > 2p_u^2\lambda(1-p_q),\end{aligned}$$

since $(1-p_q)(\lambda-1) < 1$. This yields

$$p_q(\alpha-\lambda)(p_q-p_u)+p_q(1-2p_u)p_u\lambda+2p_u^2p_q\lambda > 2p_u^2\lambda. \qquad (8.2)$$

Subtracting the LHS of (8.2) from that of (8.1), we have

$$p_up_q\lambda(1+p_u)-p_q(1-2p_u)p_u\lambda-2p_u^2p_q\lambda = p_u^2p_q\lambda > 0.$$

Hence, if (4.2) holds, $F > 0$. ■

We end by proving that the optimal affirmative action policy is feasible.

Proof of Proposition 5.6: By Step 3 of Lemma 5.1, if $(\bar\psi^h, \bar\psi^p)$ is infeasible, it must be because $\bar\psi^p$ causes a net loss to the firm. We show that this is not the case.

Let $\lambda \geq 1 + \frac{1}{1-p_u}$. Then

$$\begin{aligned} V(\pi^{p*}(\bar{\psi}^h, \bar{\psi}^p)) &= V(\pi^{p*}(\bar{\psi}^h, \bar{\psi}^p = 1) = (1-p_u)(\lambda - 1)[(\alpha - \lambda) + p_u\lambda] - p_u\lambda \\ &\geq [(\alpha - \lambda) + p_u\lambda] - p_u\lambda, \text{ since } (1-p_u)(\lambda-1) \geq 1 \\ &= \alpha - \lambda > 0. \end{aligned}$$

Now let $\lambda < 1 + \frac{1}{1-p_u}$ and $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ be decreasing in ψ_U^p. Then

$$V(\pi^{p*}(\bar{\psi}^h, \bar{\psi}^p)) = V(\pi^{p*}(\bar{\psi}^h, \psi^p = 0) = (1-p_q)^2(\alpha - \lambda)(\lambda - 1) > 0.$$

Lastly, let $\lambda < 1 + \frac{1}{1-p_u}$ and $V(\pi^{p*}(\bar{\psi}^h, \psi_U^p))$ be increasing in ψ_U^p. Then

$$V(\pi^{p*}(\bar{\psi}^h, \bar{\psi}^p)) = V(\pi^{p*}(\bar{\psi}^h, \psi^p = 1) = (1-p_u)(\lambda - 1)[(\alpha - \lambda) + p_u\lambda] - p_u\lambda.$$

Note that this outcome corresponds to the L-L equilibrium exhibited in Section 2, where $\pi_{1,1}^{h*} = \min\{1; 2 + p_u(\lambda - 1)\} = 1$. For the L-L equilibrium to exist, $\pi_{1,1}^{h*} \geq \hat{\pi}_1^h$, where

$$\hat{\pi}_1^h = \frac{1}{1 + \frac{p_q}{p_u}[\pi_{1,1}^{p*}(\alpha - \lambda + p_u\lambda) - p_u\lambda]}$$

is the belief at which the firm is indifferent between hiring an unclear signal and rejecting him. Substituting $\pi_{1,1}^{p*} = (1-p_u)(\lambda - 1)$ and transposing terms, we have

$$1 + \frac{p_q}{p_u}[(1-p_u)(\lambda - 1)(\alpha - \lambda + p_u\lambda) - p_u\lambda] \geq 1$$

$$\text{or, } (1-p_u)(\lambda - 1)[(\alpha - \lambda) + p_u\lambda] - p_u\lambda > 0.$$

Hence, the result is established. ■