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Discrimination in festival games with limited observability and accessibility☆

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ABSTRACT

This paper provides an analysis of discrimination and prejudices from the perspective of inductive game theory. We extend the festival game, originally given by Kaneko–Matsui, to include new constraints on the observability of ethnic identities and on accessible locations for players. We characterize the Nash equilibrium set, which reveals a different variety of segregation patterns and discriminatory behavior. In order to facilitate the analysis of discrimination and prejudices, we introduce a measure of discrimination, which chooses a representative equilibrium with the smallest degree of discrimination. Using this measure, we discuss various new phenomena, such as discrimination in an ethnic hierarchy; similar ethnicities as discriminated and as discriminating; and mutual discrimination. The introduction of limited observability and accessibility enables us to obtain those results.

1. Introduction

1.1. Motivations and background

Discrimination and prejudices are widespread social phenomena. Among serious ones, we have discrimination against Blacks in the US, Dalits in India, and Burakus in Japan. Also, we find other types of discrimination such as gender, political, economic, and religious. Plenty of such instances are found all over the world in different dimensions (see Marger (1991) for a comprehensive survey). We can expect to have a game/economic theoretical study of discrimination and prejudices, since game theory and economics are regarded as well developed sciences of socioeconomic phenomena.

However, if we look at the phenomena of discrimination and prejudices more seriously, we find that they serve a challenge to these disciplines. These disciplines have emphasized rational thinking and rational behavior, but in reality, we are all constrained with a lot of limitations on behavior, experiences, knowledge, and also thinking. Discrimination and prejudices have cognitive and behavioral aspects consisting of ignorance, falsities, and their adverse behavioral consequences. In order to take those issues seriously, we need to extend/develop those disciplines more.

Based on the above idea, Kaneko and Matsui (1999) started an approach to discrimination and prejudices, using a game model called the *festival game*.[1] They discussed the possibility for a player to develop a social view including prejudicial elements from his behavioral experiences. The authors called the entire theory *inductive game theory* (IGT). This approach is converse to the standard economics literature on discrimination initiated by Becker (1957) where prejudicial (mental) elements are assumed and behavior is derived (a brief survey will be given in Section 1.3).

Recently, IGT has been developed into full-dress research in Kaneko and Kline (2008a,b), which ranges from generation of experiences, accumulation of memories, inductive construction of a view, and its behavioral uses. However, these papers targeted a general theoretical development rather than its applications in specific societal contexts. This paper reconsiders the approach in Kaneko and Matsui (1999) taking these advancements into account.

We incorporate certain limitations into the approach in Kaneko and Matsui (1999). Specifically, we add two structures to the festival game:

(a) limited observability of ethnicities;
(b) accessibility constraint on each individual player.

The first gives a limited capability to distinguish some ethnicities from others, and enables us to discuss ethnic similarity and ethnic distance. The second constrains a player's trials of going to

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[1] The game model called a "festival game" was considered in Kaneko and Kimura (1992). It was a simpler strategic form game than the extensive game in Kaneko and Matsui (1999).

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other festivals, and may be interpreted as lack of experience or institutional constraints on mobility. (a) is assumed to be ethnicity-specific but (b) is individual-specific, which will be discussed in Section 2.1. With these, the theory is able to capture new phenomena of discrimination, which were not possible in Kaneko and Matsui (1999) and will be listed in Section 1.2.

In IGT, we start with the *no-knowledge assumption* that a player has no prior knowledge about the game structure.[2] Instead, some knowledge is acquired from past experiences of playing the game. As mentioned above, the full theory consists of processes of generation/accumulation of experiences and memories, inductive understanding and its behavioral uses. After repeating these processes for some time, each player reaches optimal behavior up to his perceptional and behavioral limitations. Behaviorally, this forms Nash equilibrium subject to partial understanding of the game and available actions. This interpretation of Nash equilibrium (explained in Section 2.4) differs from the *ex ante* game theory approach following Nash (1951), which assumes full understanding of the game, and from the evolutionary game theory approach (cf. Weibull, 1995), which has no scope for discussing the formation of knowledge.

We adopt Nash equilibrium from the viewpoint of IGT as the solution concept. But there are a large number of Nash equilibria, including different degrees of discrimination. We focus on the minimum degree of discrimination, and define the *measure of discrimination* to be this degree. The measure gives various interesting findings, which are discussed in Section 1.2.

1.2. Some results on discriminatory phenomena

The festival game is played by a population of players, who are only symbolically differentiated with ethnicities. The ethnicity of a player has no direct effect on available actions and payoffs. The game has two stages: in the first, each player chooses one of the available locations. In the second, he observes the configuration of ethnicities in the location, and then chooses his attitude – either friendly or unfriendly – to the other players there. If he chooses to be friendly, the payoff for him would be the mood of the festival, i.e., the number of friendly players in his location; and otherwise, the payoff would be a low threshold level. Fig. 1.1 illustrates a festival game with 4 locations and 4 ethnic groups. When all people are behaving friendly, the height of each rectangle represents the payoff to each player there.

The above situation is considered from the viewpoint of IGT. It is repeated, and each player follows some regular behavior but makes a trial deviation once in a while. These trials will give him some knowledge about social responses to his deviations. In particular, some players may change their behavior from the friendly to unfriendly action in response to a visitor from another location. We call this behavior *discrimination*. In our interpretation, discriminatory responses to other ethnicities are regarded as social conventions developed in history.

In addition to the new structures (a) and (b) mentioned in Section 1.1, this paper has two points to be emphasized:

(i) introduction of a discrimination measure; and
(ii) new findings about discriminatory phenomena using the discrimination measure and the additional structures.

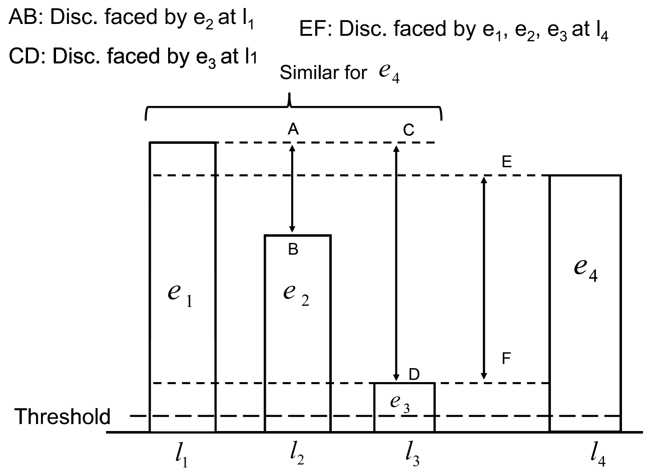

Fig. 1.1. Ethnic hierarchy in similar groups.

As mentioned in Section 1.1, a Nash equilibrium may contain some pattern of segregation, and a Nash equilibrium with segregation necessarily involves discrimination of various degrees. We focus on the minimum degree of discrimination needed to sustain Nash equilibrium, and define a measure of discrimination to be this degree.

The definition and calculation of the discrimination measure need a full study of the set of Nash equilibria. This requires some technical development before reaching its definition and the main results. It would be convenient for a reader to have an idea of such results without going through the rigorous development. Here we illustrate four results given formally in Sections 4 and 5.

In Fig. 1.1, each rectangle represents the size of population at the location and represents its mood when all players are friendly there. Thus, a player in a smaller rectangle could have an incentive to move to a larger rectangle if he met no discrimination. As we show later, Nash equilibrium requires that when a player goes to a larger festival without coethnic players, he would face discrimination from some players in the larger festival. These discriminatory responses remove the incentive to move.

The following three discriminatory phenomena can occur simultaneously in Fig. 1.1.

(1) **Ethnic hierarchy**. Consider the people of ethnicities e_1, e_2, e_3 at locations l_1, l_2, l_3 in Fig. 1.1. Those ethnicities are assumed to be distinguished among those people. To keep this location configuration, people at l_1 discriminate against those of e_2, e_3 when they come to l_1. Here, our discrimination measure states that the minimum degree of discrimination against e_2 at l_1 is given as the length *AB*, and that the corresponding degree against e_3 is the length *CD*. Hence, the necessary degrees of discrimination against minorities are reciprocal to the sizes of targeted groups. Minorities face severe discrimination following the hierarchy of group sizes.[3]

If we focus on ethnic hierarchy alone, we can obtain this result in the framework in Kaneko and Matsui (1999) with the use of the discrimination measure. However, with limited observability, it can occur simultaneously with the following.

(2) **Similar ethnicities as discriminated**. People of e_4 form the second largest group in Fig. 1.1. Suppose that they have some sociocultural distance from the other ethnic people, and that they

[2] This is not formulated as "uncertainty", which requires each player to be cognizant of the possible structures. See Kaneko and Kline (2008a) and Kaneko and Kline (in press-a,b) for this issue.

[3] This appears not to fit some cases such as apartheid in South Africa, if the sizes of the festivals are literally interpreted as defining "minories". We interpret the size of an ethnic group as the relative group advantage. In the example of Sough Africa, a numerically superior group such as the Blacks may be less advantaged than a numerically inferior group such as the Indians. See Marger (1991).

cannot distinguish between e_1, e_2, e_3. Then, when a person from l_1, l_2 or l_3 tries to go to l_4, the people in l_4 cannot tell which location he comes from. The discrimination measure states that e_1, e_2, e_3 face the same degree of discrimination given as EF. A person of e_1 from the largest group is treated in the same way as e_3 from the smallest group. Such a phenomenon is observed in Africa, where there are hierarchies of groups among them, but once they go to the US, they are treated as the same category "black".

(3) **Mutual discrimination**. People of e_4 are discriminated against in l_1, because l_1 is larger than l_4. In (2), we already observed discrimination against e_1 in l_4. Thus, we have mutual discrimination between the ethnic groups e_1 and e_4, instead of just the majority discriminating against a minority.

The discriminatory phenomena (1)–(3) can possibly occur simultaneously, which relies upon limited observability. We have another phenomenon where discrimination happens only between similar ethnicities. This can be described by extending Fig. 1.1, but we separate it for clarity.

(4) **Similar ethnicities as discriminators**. Look at Fig. 1.2, where people of ethnicity e_2' live in location l_1 as a minority. They were originally divided from the people of ethnicity e_2 due to some historical reason. The majority of e_1 cannot distinguish between e_2' and e_2, but e_2' people can distinguish e_2 from themselves. It is possible to have the segregation pattern in Fig. 1.2 as a Nash equilibrium if enough people of e_2' discriminate e_2. The minimum degree of discrimination is given as the length GH. Only a minority can keep this segregation by discriminating against their cousins.

The assumption of limited accessibility has not entered the above arguments. However, when we look at the entire IGT scenario for these phenomena, it plays an important role. For example, e_2' in (4) may have originated from e_2 by some historical isolation. Also, it restricts an individual player's experiences and generates more possibilities for his subjective view of society or prejudices. We touch upon this issue in Section 6. The possibility is shown there that the segregation pattern itself is the same as in the festival game without limited accessibility, but the underlying behavioral patterns are very different. Also, we find a richer variety of possible prejudicial views.

1.3. Contributions to the literature

The economics literature typically applies some mathematical method from micro-economics to the problem of prejudices and discrimination. Our approach has more in common with sociological and social–psychological literatures (cf., Marger (1991) and Brown (1995) respectively), but they emphasize empirical observations of prejudices/discrimination and non-mathematical theories. Since direct comparisons are difficult, we will refer to these literatures only in relevant places in the paper. Here, we briefly look at the economics literature below.

Becker's (1957) approach is the most direct application of neoclassical economics. Some prejudicial component is included in the utility function, and some behavioral consequences are derived (see Chan and Eyster (2003) and Basu (2005) for recent developments[4]). In this approach, we cannot address the question of how prejudices are formed; expressing prejudices in a utility function is, more or less, equivalent to assuming what is to be explained.

An alternative approach is the statistical discrimination theory by Arrow (1973) and Phelps (1972) (see Coate and Loury (1993) and Lang (1986) for other variants). In this approach, groups of people have different statistical distributions of productivities. An employer cannot directly see the productivity of a job candidate, but has information about the average productivity of a group. In the occasion of hiring a worker, he avoids the person from a group with a lower average productivity. Here, individual differences are ignored but only statistical information is taken. In this approach, real and perceived differences are magnified through limited perceptive abilities.

Our approach avoids conceptual difficulties involved in the above approaches, that is, prejudices or substantive differences are not assumed as primitives; but only symbolic differences are assumed. Also, the no-knowledge assumption of our approach is more compatible with the concept of prejudices, which often emerge as part of the group formation process. The statistical discrimination theory is somewhat related to our approach in that only group differences are taken but individual differences are ignored. Nevertheless, group differences are nominal in our approach, but they are real and already assumed in the minds of individuals in the statistical discrimination theory.

Lastly, this paper contributes to the development of IGT. The papers (Kaneko and Kline, 2008a,b) cited before have developed a theoretical framework for IGT. Also, an experimental study was undertaken in Takeuchi et al. (2011), specifically on prisoner's dilemma. Since the theoretical development does not touch concrete problems and the experimental study deals with a laboratory problem, by applying IGT to a specific social problem, this paper complements those studies.

2. Festival game with limited observability and accessibility

This section provides intended interpretations of basic components of the festival game, and then a mathematical formulation of it. We also give a brief idea of IGT, which helps understanding of the analysis of discrimination in terms of Nash equilibrium.

2.1. Festival game: intended interpretations

The festival game abstracts from institutional details in order to capture group formation and distribution of advantages over emerging groups. It is a highly simplified and focused model. Therefore, we should carefully explain the intended interpretations of the main components.

F1: *Players and Ethnicities*. The festival game attempts to capture interactions among a large population of players. The players are assumed to be identical with respect to available actions and payoffs, but they are divided into various *ethnicities*. By ethnicity, we refer to an attribute shared by some people, e.g., race, gender, caste, and/or language. In the festival game, ethnic differences are purely symbolic. Initially, players of the same ethnicity share no common beliefs/values. Rather, we consider the emergence of groups with some shared beliefs/values.

F2: *Locations and Festivals*. Each player chooses a location for having a festival. The group of players choosing the same location comprises the *festival* at that location. Thus, a festival is a group formed at a location. The difference between a location and a festival is analogous to that between a geographical area and a state.

F3: *Ethnicity Configuration*. Once a player comes to a location, he observes the set of ethnicities present at that location, which is called the *ethnicity configuration*. A player does not observe the number of players choosing the same location. This is interpreted as the result of a large population stated in F1 together with limited cognitive abilities of players mentioned in Section 1.1.

4 Basu (2005) formulated his approach in the context of evolutionary game theory, but the basic idea about discrimination and prejudice is the same as Becker (1957)'s.

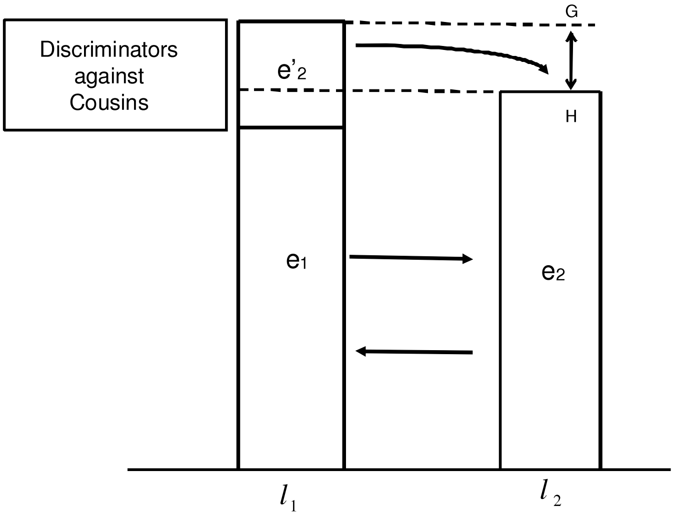

Fig. 1.2. Similar ethnicities as discriminators.

F4: *Actions*. After observing the ethnicity configuration at his location, each player chooses either a *friendly* action or an *unfriendly* action. The friendly action is interpreted as participation in the festival. On the other hand, the unfriendly action is a hostile attitude to the other players, and gives him no joy from the festival.

F5: *Mood of a festival and Payoff*. The mood of a festival is defined as the number of friendly people in the festival. This captures the relative advantage of a group over others. The payoff to a player is assumed to be the mood of the festival if he chooses to be friendly; it is a low threshold utility if he chooses to be unfriendly.

The festival game up to this point was given in Kaneko and Matsui (1999). We introduce two additional structures, already mentioned in Section 1.1.

F6: *Limited Observability*. A player may or may not distinguish between similar ethnicities. We assume that this is common for all members of an ethnic group, meaning that every member of an ethnicity has the same ability of perceiving other ethnicities. Limited observability allows us to study the effects of ethnic differences on discrimination. Distinguishability and indistinguishability among ethnicities in Fig. 1.1 are described in terms of this concept.

F7: *Limited Accessibility*. Trial deviations of players are limited. Some players do not go to other locations at all; they may be hesitant, lazy, or even do not think about other locations. By this, a player has limited experiences, which will be described by the set of accessible locations from to the location he regularly goes to. Alternatively, F7 can be interpreted as socio-political restrictions on mobility like immigration barriers or zoning laws. This interpretation gives a rich story for F6. Fig. 1.2 is an example of this sort, and we will give another example presently. Both interpretations will be used in the following.

Keeping the above interpretations of the components of the festival game in mind, we now go to its formal description.

2.2. Mathematical formulation of the festival game

We consider the repeated situation of the festival game Γ:

$$\cdots \boxed{\cdot}\boxed{\cdot}\boxed{\cdot}\boxed{\Gamma}\boxed{\cdot}\boxed{\cdot}\boxed{\cdot}\boxed{\Gamma}\boxed{\cdot}\boxed{\cdot}\boxed{\cdot} \quad \cdots \tag{2.1}$$

Here, we give a mathematical formulation of Γ, and formulate the Nash equilibrium in Section 2.3, which is interpreted as a stationary state in (2.1). In Section 2.4, a brief interpretation of the Nash equilibrium is discussed from the viewpoint of IGT.

The festival game Γ is played by players $1, \ldots, n$ of various ethnicities. The set of players is denoted by $N = \{1, \ldots, n\}$ and the set of ethnicities is given as $\{e_1, \ldots, e_S\}$. Let $e(\cdot)$ be a function from N to $\{e_1, \ldots, e_S\}$. The value $e(i)$ is the *ethnicity* of player i. When $e(i) = e(j)$, we call i and j *coethnic players*. There are T locations available for festivals, to one of which each player will go.

The game Γ has two stages:

The first stage (location choice). Every player i simultaneously chooses a location $f_i = l$ from the available locations $L_0 := \{l_1, \ldots, l_T\}$.

The second stage (choices of attitudes). Player i goes to location $f_i = l$, and observes the ethnicity configuration of location l. Using this observation, he chooses either *friendly action* 1 or *unfriendly action* 0.

The choice of player i in the first stage is denoted by $f_i \in L_0$. Thus, the choices of n players in the first stage are expressed by a vector $f = (f_1, \ldots, f_n)$, which we call a *location configuration*. In the second stage, player i observes the *ethnicity configuration for player* i at location f_i, which is defined by

$$E_i(f) = \{e(j) : f_j = f_i \text{ and } j \neq i\}. \tag{2.2}$$

This is the set of ethnicities observed by player i at location f_i. It is assumed that player i neither identifies each individual player nor does he observe the number of players from each ethnicity at f_i. This is in the same spirit as F6; limited observability will be presently formulated.

Also, we define, for $l \in L_0$,

$$E^l(f) = \{e(j) : f_j = l \text{ and } j \in N\}. \tag{2.3}$$

This is the *ethnic configuration* at location l from the *objective* point of view. If i with $f_i = l$ is the only player of ethnicity $e(i)$ at l, then $E_i(f) = E^l(f) - \{e(i)\}$; and if at least two players coethnic to $e(i)$ are at l, then $E_i(f) = E^l(f)$. We will avoid the former case by assuming one condition on $f = (f_1, \ldots, f_n)$.

As explained in F6, we introduce the additional structure called limited observability of ethnicities.

Limited Observability. We introduce an *equivalent relation* $\sim_e$ $(e = e_1, \ldots, e_S)$ over the set $\{e_1, \ldots, e_S\}$. The expression $e' \sim_e e''$ means that any player of e does not distinguish between ethnicities e' and e''. The negation of $e' \sim_e e''$ is denoted by $e' \nsim_e e''$, which means that the people of e can distinguish between ethnicities e' and e''.

For example, when the ethnic distances from e to e' and e'' are large, the players of e may treat e' and e'' as the same ethnicities, but the players of e' and e'' can distinguish between themselves. This is the example of ethnic groups in Africa and people in the US mentioned in Section 1.2.

We denote the equivalence class of $\sim_e$ including e' by $[e']_e := \{e'' : e'' \sim_e e'\}$. For a subset E of $\{e_1, \ldots, e_S\}$, we define the quotient set $E/\sim_e := \{[e']_e : e' \in E\}$. When player i of e sees the ethnicity configuration E, he actually perceives $E/\sim_e$.

We now introduce a limitation over accessible locations stated in F7.

Limited Accessibility. We restrict the location choice of player i to a nonempty subset L_i of L_0. The set L_i consists of the locations that player i has experienced and is currently remembering. We call the set $\mathcal{L} = \{L_i\}_{i \in N}$ an *accessibility structure*. We denote the festival game with an accessibility structure $\mathcal{L}$ by $\Gamma(\mathcal{L})$.

Strategies. An L_i*-strategy* for player i of e is given as a pair $\sigma_i = (f_i, r_i)$ consisting of a location choice $f_i \in L_i$ and a function $r_i : 2^{\{e_1,\ldots,e_S\}} \to \{0, 1\}$ with the requirement that for all $E, E' \in 2^{\{e_1,\ldots,e_S\}}$,

$$E/\sim_e = E'/\sim_e \quad \text{implies} \quad r_i(E) = r_i(E'). \tag{2.4}$$

Condition $f_i \in L_i$ is a location choice constraint, and (2.4) is an action choice constraint, stating that the choice $r_i(E) = 1$ or $r_i(E) = 0$ depends upon the ethnicity configuration E up to player i's perceptional ability. Let $\Sigma_i(L_i)$ be the set of all L_i-strategies for i.

We say that $\sigma = (f, r) = ((f_1, \ldots, f_n), (r_1, \ldots, r_n))$ is an $\mathcal{L}$*-profile* iff $\sigma_i = (f_i, r_i)$ is an L_i-strategy for all $i \in N$. By $\Sigma(\mathcal{L})$, we denote the set of $\mathcal{L}$-profiles $\sigma = (\sigma_1, \ldots, \sigma_n)$.

Mood and Payoffs. When the players behave according to a profile $\sigma = (f, r) \in \Sigma(\mathcal{L})$, the payoff to each player i is determined by the mood for him at f_i and his attitude determined by r_i. Under a strategy profile $\sigma = (f, r)$, the *mood* of festival l is defined to be the total number of friendly people at location l:

$$m_l(\sigma) = \sum_{f_j = l} r_j(E_j(f)). \tag{2.5}$$

Now we define the *payoff function* of player i by:

$$H_i(\sigma) = \begin{cases} m_l(\sigma) & \text{if } r_i(f) = 1 \\ m_0 & \text{if } r_i(f) = 0, \end{cases} \tag{2.6}$$

where the *threshold utility* m_0 is a non-integer real number. Definition (2.6) means that when he takes a friendly action, his payoff is the mood of his festival, but when he acts unfriendly, his payoff becomes the threshold utility m_0. The threshold m_0 may be interpreted as the utility from staying at home. The non-integerness of m_0 avoids tie-situations between unfriendly and friendly actions.

Now, we have the festival game with limited observability and accessibility as the triple $\Gamma(\mathcal{L}) = \langle N, \{\Sigma_i(L_i)\}_{i \in N}, \{H_i\}_{i \in N} \rangle$.

Example 2.1. In Fig. 2.1, e_2 people are split into two locations, and the dotted line is a barrier preventing people going from l_2 to l_1, but people in l_1 can access l_2. Here, we have $L_i = \{l_1, l_2\}$ for all i in l_1, but $L_i = \{l_2\}$ for all i in l_2. As mentioned above, this barrier may be interpreted as no trials by e_2 people or an institutional constraint against new immigration of e_2 people to l_1.

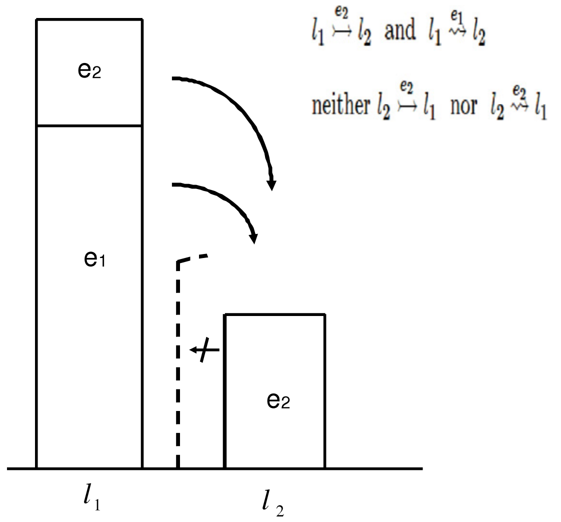

Fig. 2.1. Ethnic barrier.

Limited observability and limited accessibility may sometimes show similar phenomena. In Fig. 1.2, it may be the case that $L_i = \{l_1, l_2\}$ for all i in l_1 and l_2. However, e_2' people at location l_1 can distinguish themselves from e_2 people from l_2, while e_1 people cannot. This distinguishability may prevent the migration of e_2 to l_1, due to discrimination only by e_2'. Thus, we have similar segregation patterns, but the underlying reasons for them are different. This implies that an appropriate policy to abolish such segregations differs in the two cases — in the case of Fig. 2.1, removing the barrier is the target; and in the case of Fig. 1.2, preventing discriminatory behavior with education or affirmative action should be the target.

Comment on ethnicity-specific observability. It may be wondered why L_i is player-specific while the indistinguishability relation $\sim_e$ is ethnicity-specific. This comes from the reason that L_i describes his own previously taken experiences, while $\sim_e$ describes the passive observation ability, common to the players of an ethnicity. It would be possible to assume that the latter is also player-specific. However, we focus on the common case for simplicity.

2.3. Nash equilibrium and basic properties

We say that an $\mathcal{L}$-profile σ is a *Nash equilibrium* in $\Gamma(\mathcal{L})$ iff for all $i \in N$,

$$H_i(\sigma) \geq H_i(\sigma_{-i}, \sigma_i') \quad \text{for all } \sigma_i' \in \Sigma_i(L_i). \tag{2.7}$$

We discuss the interpretation of Nash equilibrium from the IGT point of view in Section 2.4. Here, we repeat a small observation given in Kaneko and Matsui (1999): we can replace (2.7) by maximization over a narrower class of strategies: In a deviation, a response in the second stage does not need to take the ethnicity configuration into account. For completeness, we give a proof.

Lemma 2.1. *An $\mathcal{L}$-profile σ is a Nash equilibrium in $\Gamma(\mathcal{L})$ if and only if for any $i \in N$,*

$$H_i(\sigma) \geq H_i(\sigma_{-i}, (l, \delta_i)) \quad \text{for all } (l, \delta_i) \in L_i \times \{0, 1\}. \tag{2.8}$$

Here δ_i is regarded as a constant function over $2^{\{e_1,\ldots,e_S\}}$ taking a value from $\{0, 1\}$.

Proof. It suffices to consider the *if*-part. Suppose that player i takes a new strategy $\sigma_i' = (l, r_i') \in \Sigma_i(L_i)$. Player i moves to a location l and observes the ethnicity configuration $E_i(f_{-i}, l) = E^l(f)$. In l, his action is $r_i'(E_i(f_{-i}, l))$. Here, his payoff is determined by the action $r_i'(E_i(f_{-i}, l))$; the functional structure of r_i' is irrelevant but the function value is relevant. Let $\delta_i = r_i'(E_i(f_{-i}, l))$. Then (l, δ_i) plays the same role as $\sigma_i' = (l, r_i')$, and gives the same payoff. Thus, $\Sigma_i(L_i)$ in (2.7) is restricted to $L_i \times \{0, 1\}$. □

We note that σ_i itself may not belong to $L_i \times \{0, 1\}$. Simple behavior of taking actions is enough for deviations, but the response structure described in $r = (r_1, \ldots, r_n)$ is essential to attain the stability of a Nash equilibrium including segregation and discriminatory behavior.

We state one more small observation.

Lemma 2.2 (*Active Festivals*). *Let $\sigma = ((f_1, r_1), \ldots, (f_n, r_n))$ be a Nash equilibrium in $\Gamma(\mathcal{L})$. If $r_i(E_i(f)) = 1$ for some $i \in N$, then $r_j(E_j(f)) = 1$ for all j with $f_j = f_i$.*

Proof. Let $r_i(E_i(f)) = 1$ for some $i \in N$. Then, $H_i(\sigma) = m_l(\sigma) \neq m_0$ since $m_l(\sigma)$ is an integer but m_0 is not. If $H_i(\sigma) = m_l(\sigma) < m_0$, player i would get m_0 by switching to unfriendly action 0, which is impossible since σ is a Nash equilibrium. Hence, $H_i(\sigma) = m_l(\sigma) > m_0$. Thus, every player at l should take friendly action 1. □

In the following, we assume the two conditions on an $\mathcal{L}$-profile $\sigma = (f, r)$.

Condition A (*Active Festivals*). For all $l \in L_0$, $r_i(E_i(f)) = 1$ for some i with $f_i = l$.

Condition M (*Multiple Coethnic Players*). For each $e = e_1, \ldots, e_S$, the number of players of e at each $l \in L_0$ is 0 or more than 1.

By Condition A and Lemma 2.2, if $\sigma = (f, r)$ is a Nash equilibrium, all locations have active festivals. By Condition M, $E_i(f)$ of (2.2) coincides with $E^l(f)$ of (2.3) with $f_i = l$. These assumptions simplify our analysis.

2.4. Nash equilibrium in $\Gamma(\mathcal{L})$ from the IGT point of view

We adopt the Nash equilibrium concept in order to analyze the behavior of people in the festival game $\Gamma(\mathcal{L})$, but its interpretation is made from the IGT point of view, rather than as the *ex ante* decision making by rational players. Here, we give a brief account of the scenario of IGT discussed in Kaneko and Kline (2008a).

IGT consists of four stages:

(i) trials/errors generating short-term memories for a player;
(ii) transformation of short-term memories into long-term memories and their accumulation;
(iii) inductive derivation of a personal view from the accumulated long-term memories;
(iv) use of a personal view for decision making and behavior revising.

Then, a player behaves in the game following the behavior revised in (iv), and starts new trials/errors. Thus, the cycle restarts from (i) again.

As discussed in Kaneko and Kline (2008a), a strategy profile which is stationary through those stages is a Nash equilibrium. In this paper, those four stages are compressed and only the resulting Nash equilibrium is considered. Still, some basic ideas of these stages are relevant for an understanding of what we are doing. We give a brief account of the relevant part.

In the recurrent situation described in (2.1), let an $\mathcal{L}$-profile $\sigma = (f, r)$ be temporarily adopted by the players. It is very basic that players do not know the structure of the game, stated as the no-knowledge assumption, but follow their behavior patterns. They make some deviations as trials/errors to get information about the responses of other people. We assume that only a small number of players make trial deviations on the locations in L_i at one time; specifically, we consider only unilateral deviations. Thus, the memories from experiences are

$$\mathcal{E}_i(\sigma) = \{[(l, \delta_i), E_i(f_{-i}, l), H_i(\sigma_{-i}, (l, \delta_i))] : l \in L_i \text{ and } \delta_i = 0, 1\}. \quad (2.9)$$

The values $E_i(f_{-i}, l)$ and $H_i(\sigma_{-i}, (l, \delta_i))$ are listed, but they are still unknown to player i as functions.

Having the set of experiences $\mathcal{E}_i(\sigma)$, player i finds a causal relationship (or correlation) from (l, δ_i) to payoff value $H_i(\sigma_{-i}, (l, \delta_i))$. Then, he chooses a (l, δ_i) from $L_i \times \{0, 1\}$ to maximize $H_i(\sigma_{-i}, (l, \delta_i))$. He modifies the corresponding part of his behavior σ_i with (l, δ_i). Now, he has the modified strategy σ_i' and brings it to the recurrent situation in (2.1). This corresponds to the stages (iii) and (iv) above.

An $\mathcal{L}$-profile σ which is stationary through the above revision process for all players is a Nash equilibrium in $\Gamma(\mathcal{L})$. Kaneko and Kline (2008a) gave a full scenario of stages (i)–(iv) in the general context of extensive games, and Kaneko and Matsui (1999) discussed possible personal views from $\mathcal{E}_i(\sigma)$. In Section 6, we will briefly discuss possible views in the festival game with an accessibility structure $\mathcal{L}$.

The response part r_i of $\sigma_i = (f_i, r_i)$ for each i is also regarded as emerging in the past. Sometimes, it has come as a custom or tradition in the community by mimicking other people. In a Nash equilibrium with segregation, it prevents the incentive for people from smaller festivals to come his festival, in which sense it is essential for this paper. It is an important point that such behavior has been emerging spontaneously rather than chosen by conscious decision making.

Effects of limited observability and accessibility on nash equilibrium. Limited observability narrows down the set of Nash equilibria, since it limits responses of players to a visiting player, which may be observed in the result (2) of Section 1.2. On the other hand, limited accessibility may increase the set of Nash equilibria, for examples, one ethnic group may be divided into several locations. In Section 6, we will give examples where $\mathcal{L}$ limits strongly players' mobilities but does not affect the Nash equilibrium outcomes.

In Section 6, it is shown that limited accessibility changes possible prejudicial beliefs. Besides this, it does not play explicit roles in the following analysis of discrimination itself. However, it is indispensable for treatments of several discriminatory results such as asymmetric treatments of similar ethnicities. Also, it is important for coherent understanding of Nash equilibrium from the IGT point of view, since with accessibility structure $\mathcal{L}$, we can assume that only a small portion of the population make trials/errors, which will be discussed in Section 6.

Comment on subgame perfection. If we additionally require *subgame perfection/sequential rationality* (Selten, 1975), all players would choose the same location in equilibrium without any discrimination. Subgame perfection in the festival game requires payoff maximization in response to a visiting player from another location. However, this needs a further deviation of each responder, e.g., from the unfriendly to friendly action, which must have a small frequency conditional upon the original trial deviation of a visiting player. This needs too many repetitions of the game together with the assumption of strong memory ability for a player. See Kaneko and Matsui (1999), Section 4.3 for a detailed argument.

3. Nash equilibria of the festival game $\Gamma(\mathcal{L})$

In Section 3.1, we give a general characterization of the equilibrium set. In Section 3.2, we study the set of equilibria relative to a fixed location configuration. These studies will be used for the introduction of a measure of discrimination in Section 4. A map of what we do in this and the next two sections is given in Fig. 3.1.

3.1. Characterization of Nash equilibria

With the additional structures $\{\sim_e\}_e$ and $\mathcal{L} = \{L_i\}_{i \in N}$, we find new phenomena in equilibrium relative to Kaneko and Matsui (1999). To investigate such phenomena, we introduce two binary relations over locations, and characterize the entire set of Nash equilibria using these relations. In this subsection, the role of $\{\sim_e\}_e$ is covert, but it will be made explicit in Section 3.2.

Let $\sigma = ((f_1, r_1), \ldots, (f_n, r_n))$ be an $\mathcal{L}$-profile. Consider an ethnicity e and two distinct locations l, l' with $f_i = l$ for some i with $e(i) = e$. Then we have two cases:

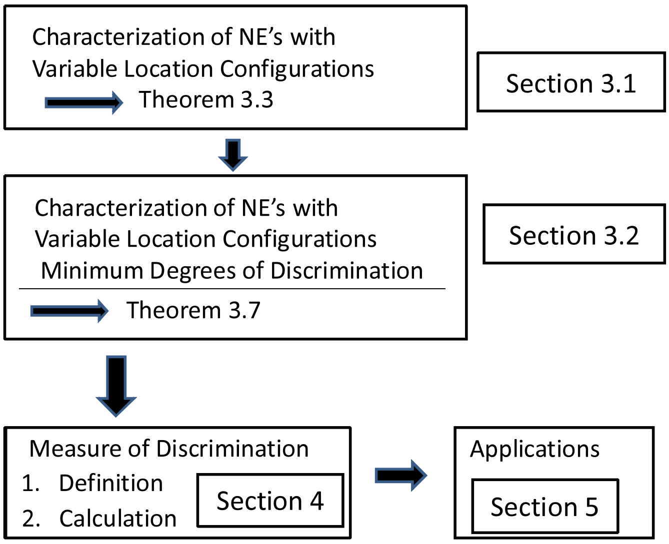

Fig. 3.1. Map of Sections 3–5.

A: no players of ethnicity e at l can access l', i.e., $l' \notin L_i$ for all i with $f_i = l$;

B: some player i with $e(i) = e$ at l can access l', i.e., $l' \in L_i$; and either
  B1: some player j coethnic to i is already at l'; or
  B2: no coethnic players are at l'.

Case A is irrelevant for the consideration of Nash equilibrium. Case B is relevant and divided into two subcases B1 and B2. These subcases should be treated differently, since the presence of player i at l' is not observed in B1, but it may be observed in B2.

These cases are mathematized as follows: we define two relationships $\overset{e}{\rightarrowtail}$ and $\overset{e}{\rightsquigarrow}$ over $\{l_1, \ldots, l_T\}$ by

(*Coethnic Players*). $l' \overset{e}{\rightarrowtail} l$ iff B and B1 hold;

(*No Coethnic Players*). $l' \overset{e}{\rightsquigarrow} l$ iff B and B2 hold.

When $l \overset{e}{\rightarrowtail} l'$, player i in B can go to l' without being noticed at l'. On the other hand, when $l \overset{e}{\rightsquigarrow} l'$, the presence of player i may be noticed by some players at l. We also write $l \rightarrowtail l'$ (respectively, $l \rightsquigarrow l'$) when $l \overset{e}{\rightarrowtail} l'$ ($l \overset{e}{\rightsquigarrow} l'$) for some $e \in \{e_1, \ldots, e_S\}$.

In Fig. 2.1, a player at l_1 can access l_2. Here, since some players of e_2 are already in l_2, we have $l_1 \overset{e_2}{\rightarrowtail} l_2$. On the other hand, since no players of e_1 are at l_2, the relation $l_1 \overset{e_1}{\rightsquigarrow} l_2$ holds. Since people in l_2 cannot access l_1, neither $l_2 \overset{e_2}{\rightarrowtail} l_1$ nor $l_2 \overset{e_2}{\rightsquigarrow} l_1$ holds.

For a profile to be a Nash equilibrium, we need to eliminate the incentive for migrating to a location with and without coethnic players. We have the following theorem.

Theorem 3.1 (*Characterization of Nash Equilibria*). *Let $\sigma = (f, r)$ be an $\mathcal{L}$-profile of strategies satisfying Conditions A and M. Then, σ is a Nash equilibrium if and only if the following* (1) *and* (2) *hold: let $l, l' \in L_0$ and $e \in \{e_1, \ldots, e_S\}$. Then*

(1) *(Coethnic Players). If $l \overset{e}{\rightarrowtail} l'$, then $m_l(\sigma) > m_{l'}(\sigma)$.*

(2) *(No Coethnic Players). If $l \overset{e}{\rightsquigarrow} l'$ and i is a player of e at l with $l' \in L_i$, then $m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1))$.*

Proof. (*Only-If*).

(1) Let $l \overset{e}{\rightarrowtail} l'$. Then, $f_i = l$ and $f_j = l' \in L_i$ for some coethnic players i, j of e. By Condition A and Lemma 2.2, $r_i(f_i, E_i(f)) = 1$, and the payoff to player i under σ is $H_i(\sigma) = m_l(\sigma)$. If i moves to l', his payoff will be $H_i(\sigma_{-i}, (l', 1)) = m_{l'}(\sigma) + 1$, since his presence is not observed at l' by $l \overset{e}{\rightarrowtail} l'$ and Condition M. Since σ is a Nash equilibrium, we have $H_i(\sigma) \geq H_i(\sigma_{-i}, (l', 1))$, i.e., $m_l(\sigma) > m_{l'}(\sigma)$.

(2) Suppose that $l \overset{e}{\rightsquigarrow} l'$ and i is at l with $e(i) = e$ and $l' \in L_i$. Since he has no coethnic players at l', his presence at l' may be observed. Therefore, his presence may induce a new mood $m_{l'}(\sigma_{-i}, (l', 1))$. Since σ is a Nash equilibrium, we have $m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1))$.

(*If*): We prove that σ is a Nash equilibrium in $\Gamma(\mathcal{L})$. By Lemma 2.1(1), it suffices to show that for any player i and any trial $(f_i, \delta_i) \in L_i \times \{0, 1\}$, $H_i(\sigma) \geq H_i(\sigma_{-i}, (f_i, \delta_i))$. Let $e(i) = e$, $f_i = l$ and consider $l' \in L_i$ with $l' \neq l$.

Now, suppose $l \overset{e}{\rightarrowtail} l'$. Then the presence of player i at l' is not observed at all, where Condition M is used. By (1), $m_l(\sigma) > m_{l'}(\sigma)$. Thus, $H_i(\sigma) = m_l(\sigma) \geq m_{l'}(\sigma) + 1 = H_i(\sigma_{-i}, (l', 1))$. Using Condition A, $H_i(\sigma) = m_l(\sigma) > m_0 = H_i(\sigma_{-i}, (l', 0))$.

Suppose $l \overset{e}{\rightsquigarrow} l'$. Then by (2), $H_i(\sigma) = m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1)) = H_i(\sigma_{-i}, (l', 1))$. Finally, using Condition A, $H_i(\sigma) = m_l(f) > m_0 = H_i(\sigma_{-i}, (l', 0))$. □

Limited observability $\{\sim_e\}_e$ is covert in Theorem 3.1(2). Specifically, the induced mood $m_{l'}(\sigma_{-i}, (l', 1))$ depends upon $\{\sim_e\}_e$. This will be analyzed later.

Both $\rightarrowtail$ and $\overset{e}{\rightarrowtail}$ are asymmetric by Theorem 3.1(1) and irreflexive by definition. But, they may not be transitive, since $\rightarrowtail$ may depend upon e and since even if $l \overset{e}{\rightarrowtail} l'$ and $l' \overset{e}{\rightarrowtail} l''$, all players of ethnicity e at l may be prohibited to access l''. Similar observations hold for $\rightsquigarrow$ and $\overset{e}{\rightsquigarrow}$.

Theorem 3.1 shows that the Nash equilibria may involve the partial or complete segregation of different ethnicities. When a Nash equilibrium involves such segregation and some accessibility is allowed, it is sustained by discriminatory behavior of some people. The relation between segregation and discrimination will be clearer in the next subsection.

Finally, we give two small implications from Theorem 3.1. Let $\sigma = (f, r)$ be a Nash equilibrium in $\Gamma(\mathcal{L})$, and let $\{l_1, \ldots, l_k\}$ be a sequence of locations satisfying $l_t \rightarrowtail l_{t+1}$ for $t = 1, \ldots, k-1$. Then

$$m_{l_1}(\sigma) > m_{l_2}(\sigma) > \cdots > m_{l_k}(\sigma). \tag{3.1}$$

When locations are connected as a chain of coethnic players, possibly with different ethnicities, we have a hierarchy of festival sizes. Secondly, let i and j be coethnic players. Then

$$f_j \in L_i \quad \text{and} \quad f_i \in L_j \quad \text{if and only if} \quad f_i = f_j. \tag{3.2}$$

That is, coethnic players would get together to one location if there are no constraints on their mobilities.

3.2. Nash equilibria with a given location configuration

From the viewpoint of IGT, a Nash equilibrium corresponds to a particular state of society together with the history that led to the state. The state includes a location configuration f, which is interpreted as a pattern of segregation. Now, we focus on this location configuration f and study the multiple Nash equilibria compatible with it. Let $\Xi(f, \mathcal{L})$ be the set of Nash equilibria $\sigma = (f, r)$ in $\Gamma(\mathcal{L})$ with a fixed $f = (f_1, \ldots, f_n)$ satisfying Conditions A and M. This set will be used in the definition of a measure of discrimination in Section 4.

By Condition A and Lemma 2.2, the mood $m_l(\sigma)$ is invariant over all Nash equilibria σ in $\Xi(f, \mathcal{L})$, and is the number of players, $|\{j : f_j = l\}|$, at location l. Defining $m_l(f) := |\{j : f_j = l\}|$ for $l \in L_0$, we have

$$m_l(f) = m_l(\sigma) \quad \text{for any } \sigma \in \Xi(f, \mathcal{L}). \tag{3.3}$$

Using this, we give conditions for the nonemptiness of $\Xi(f, \mathcal{L})$.

Theorem 3.2 (*Nonemptiness Criterion*). *Let $f = (f_1, \ldots, f_n)$ be any location configuration compatible with $\mathcal{L}$. Then $\Xi(f, \mathcal{L})$ is nonempty if and only if for any $l, l' \in L_0$,*

(1) *if $l' \overset{e}{\rightarrowtail} l$, then $m_{l'}(f) > m_l(f)$;*
(2) *if $l' \overset{e}{\rightsquigarrow} l$, then the original mood at l' is larger than the number of players at l who cannot perceive the presence of e, i.e.,*

$$m_{l'}(f) > |\{j : f_j = l \text{ and } (E_j(f) \cup \{e\})/\sim_{e(j)} \\ = E_j(f)/\sim_{e(j)}\}|. \tag{3.4}$$

Proof. (*Only If*). Let $\sigma \in \Xi(f, \mathcal{L})$. Since all players are friendly in equilibrium by Condition A and Lemma 2.2, any location is active. Let $l \overset{e}{\rightarrowtail} l'$. Then, we have $m_l(f) > m_{l'}(f)$ by Theorem 3.1(1), which is assertion (1).

Consider (2). Let $l' \overset{e}{\rightsquigarrow} l$. The right term of (3.4) is the number of players finding no difference in ethnicities at l with the presence of e and keeping friendly actions to the presence of e. This is the minimum mood possibly induced by e. If (3.4) does not hold, we cannot prevent a player i at l' with $l \in L_i$ and $e(i) = e$ from going to location l. This is impossible since $\sigma \in \Xi(f, \mathcal{L})$. Hence, we have (3.4).

(*If*). For all $i \in N$, we define $r_i : 2^{\{e_1,\ldots,e_S\}} \to \{0, 1\}$ by

$$r_i(E) = \begin{cases} 1 & \text{if } E/\sim_{e(i)} = E_i(f)/\sim_{e(i)} \\ 0 & \text{otherwise.} \end{cases} \tag{3.5}$$

It suffices to show that $\sigma = (f, r)$ is a Nash equilibrium in $\Gamma(\mathcal{L})$. If $l' \overset{e}{\rightarrowtail} l$, then $m_{l'}(f) > m_l(f)$ by (1). This means that any player i at l' with $e = e(i)$ is better to stay at l'. Suppose that $l' \overset{e}{\rightsquigarrow} l$ and a player at l' with $e = e(i)$ goes to l. By (3.5), all players at l who find the presence of player i act unfriendly. The mood induced at l is the right term of (3.4), possibly plus 1, and does not exceed $m_{l'}(f)$. Hence, σ is a Nash equilibrium and $\Xi(f, \mathcal{L}) \neq \emptyset$. □

When a player of e can move from l' to l without being found, stability requires the inequality in (1). Without limited observabilities $\{\sim_e\}_e$, every player at l could observe the presence of ethnicity e in (2), i.e.,the right-hand side of (3.4) is 0, and thus (2) could be redundant. However, with $\{\sim_e\}_e$, many players may not perceive the presence of e, and hence we need (2). This theorem shows the difference generated by the introduction of $\{\sim_e\}_e$.

Now, we define the notion of conditional mood $m_l(\sigma \mid e)$ when a player i with $e(i) = e$ comes to l and acts friendly, that is,

$$m_l(\sigma \mid e) = m_l(\sigma_{-i}, (l, 1)). \tag{3.6}$$

Though $m_l(\sigma \mid e)$ is defined by a move of player i to l, it depends only upon σ and e.

The set $\Xi(f, \mathcal{L})$ consists of many Nash equilibria, and its multiplicity corresponds to the different degrees of discriminatory responses to a visiting player. In the next section, we introduce a measure of discrimination degrees. We finish this section with a small observation about the case of $l' \overset{e}{\rightarrowtail} l$. Since some coethnic player of i is already in l, the presence of a player of e is not observed, and hence, $m_l(\sigma \mid e) = m_l(\sigma) + 1$. No discrimination is induced, and the mood is invariant over all Nash equilibria in $\Xi(f, \mathcal{L})$. We state this fact as a lemma.

Lemma 3.3 (*No Discrimination to Coethnic Players*). *Let $l' \overset{e}{\rightarrowtail} l$. Then, $m_l(\sigma \mid e) = m_l(\sigma' \mid e) = m_l(\sigma) + 1$ for any $\sigma, \sigma' \in \Xi(f, \mathcal{L})$.*

Thus the case of $l' \overset{e}{\rightarrowtail} l$ is inessential for the consideration of discrimination. In the case of $l' \overset{e}{\rightsquigarrow} l$, a variety of discriminatory responses may be observed. This is our target, and a measure of discrimination will help us study this case.

4. Measure of discrimination

Here, we define a measure of discrimination involved in Nash equilibrium relative to a given location configuration f, and then analyze its properties. As discussed in Section 3, there are many Nash equilibria $\sigma = (f, r)$ with a fixed $f = (f_1, \ldots, f_n)$; accordingly, the degree of discrimination depends upon a Nash equilibrium. Nevertheless, it is useful to have a unidimensional measure indicating the degree of discrimination necessarily occurring. Our measure gives the minimum degree of discrimination needed to sustain f as an equilibrium configuration. We assume $\Xi(f, \mathcal{L}) \neq \emptyset$ throughout the following.

Let $l \in L_0$. We define the set of *relevant ethnicities* for location l by

$$E^l(f, \mathcal{L}) = \{e : l' \overset{e}{\rightarrowtail} l \text{ for some } l'\} \cup \{e : l' \overset{e}{\rightsquigarrow} l \text{ for some } l'\}. \tag{4.1}$$

This is the set of ethnicities which have possibly visited location l. Since the latter part $\{e : l' \overset{e}{\rightsquigarrow} l \text{ for some } l'\}$ is essential, we denote this part by $E^l_n(f, \mathcal{L})$.

The *discrimination measure* $d_f(e \mid l)$ is defined over $E^l(f, \mathcal{L})$: for any $e \in E^l(f, \mathcal{L})$,

$$d_f(e \mid l) = \min_{\sigma \in \Xi(f, \mathcal{L})} [m_l(f) - (m_l(\sigma \mid e) - 1)]. \tag{4.2}$$

It is intended to be the minimum discrimination degree faced by a player of e at l. The difference $m_l(f) - (m_l(\sigma \mid e) - 1)$ is the pure change of the mood at l caused by the presence of a player of e; the last -1 eliminates the effect of his own participation on the mood. Thus, it is the number of players at l who took the discriminatory (unfriendly) actions toward the presence of e. So, $m_l(f) - (m_l(\sigma \mid e) - 1) \geq 0$. We take the minimum of such differences over all possible Nash equilibria in $\Xi(f, \mathcal{L})$. Thus, $d_f(e \mid l)$ is the degree of discrimination that will necessarily occur at l against ethnicity e in any Nash equilibrium in $\Xi(f, \mathcal{L})$.

Let us look at Fig. 1.1. The location configuration in Fig. 1.1 can be sustained by many Nash equilibria. In one equilibrium, all players at l_1 may discriminate against a visiting player from l_2; in another equilibrium, only some players at l_1 may discriminate. The value $d_f(e_2 \mid l_1)$ is the minimum degree of discrimination for such equilibria, which is the height AB. Similarly, the value $d_f(e_3 \mid l_1)$ is CD.

These observations look straightforward in the example of Fig. 1.1, but a rigorous calculation of $d_f(e \mid l)$ may not in a general case, since it needs to find an equilibrium supporting the minimum degree of discrimination. We give several assertions so as to calculate the value $d_f(e \mid l)$ in various cases, which are enough for the results mentioned in Section 1.2. A development of a general theory of calculation of $d_f(\cdot \mid \cdot)$ is open.

We introduce one definition to save mathematical expressions. Let E be a set of ethnicities and e an ethnicity. We say that e is *distinguished from E* by e'' iff $e \not\sim_{e''} e'$ for all $e' \in E$, which is denoted by $e \not\sim_{e''} E$. Also, for a set F of ethnicities, we write $e \not\sim_F E$ iff $e \not\sim_{e''} E$ for all $e'' \in F$. This means that e and each e' of E are distinguished by all players of ethnicities in F.

When a player of ethnicity e comes to location l, he may be confused with some other ethnicities. The following condition states that e is distinguished from the other relevant ethnicities at location l:

$$e \not\sim_{E^l(f)} E^l(f, \mathcal{L}) \cup E^l(f) - \{e\}. \tag{4.3}$$

Recall that $E^l(f)$ is defined in (2.3). We call this the *no-confusion assumption for e at l*. Under this condition, if $e \in E^l(f, \mathcal{L})$ and $l' \overset{e}{\rightsquigarrow} l$ for some l', then e is distinguished from any other ethnicities in $E^l(f, \mathcal{L})$ as well as $E^l(f)$.

We start with simple cases.

Lemma 4.1 (*No Discrimination*). *Let $l, l' \in L_0$, and $e \in E^l(f, \mathcal{L})$.*

(1) *If $e \notin E^l_n(f, \mathcal{L})$, then $d_f(e \mid l) = 0$.*
(2) *Let $l' \overset{e}{\rightsquigarrow} l$. If $m_{l'}(f) \le m_l(f)$, then $d_f(e \mid l) > 0$.*
(3) *Let $l' \overset{e}{\rightsquigarrow} l$ and (4.3) for e at l. If $m_{l'}(f) > m_l(f)$, then $d_f(e \mid l) = 0$.*

Proof. (1): Since $m_l(\sigma \mid e) = m_l(f) + 1$ for all $\sigma \in \Xi(f, \mathcal{L})$ by Lemma 3.3, we have $d_f(e \mid l) = 0$.

(2): Let $m_{l'}(f) \le m_l(f)$. If $d_f(e \mid l) = 0$, there is a Nash equilibrium σ such that ethnicity e is not discriminated against at l, which implies that by moving to l, he would get his payoff $m_l(f) + 1 > m_{l'}(f)$, a contradiction. Hence, $d_f(e \mid l) > 0$.

(3): Let $m_{l'}(f) > m_l(f)$. Consider a player i with $f_i = l'$ and $e(i) = e$. When he goes to l, he is distinguished from any other visitors. Hence, the players at l can be friendly to him without changing their attitudes to his presence independently of other ethnicities. Since $m_{l'}(f) > m_l(f)$, player i has no incentive to come to l even with no discrimination against him. This can be a Nash equilibrium. Hence, $d_f(e \mid l) = 0$. □

The first simply states that if a visiting player of e has a coethnic player at l, he will face no discrimination, since i's presence is not observed by the players at l. Second, if he has no coethnic players, and if the mood at l is as good as at l', he would necessarily face discrimination. Third, under the assumption that ethnicity e is not confused with any other ethnicities at l, then if l' is larger than l, then e is not discriminated at l.

Now, we consider the case of Lemma 4.1(2) more precisely.

Theorem 4.2 (*Non-Confusion*). *Let $l \in L_0$. Suppose that $m_{l'}(f) < m_l(f)$ for some l' with $l' \overset{e}{\rightsquigarrow} l$, and also the no-confusion condition (4.3) for e at l. Then*

$$d_f(e \mid l) = m_l(f) - \min_{l' \overset{e}{\rightsquigarrow} l}(m_{l'}(f) - 1), \tag{4.4}$$

where l' is the variable for the minimization operator.

Proof. Let $m_{l^o}(f) = \min_{l' \overset{e}{\rightsquigarrow} l} m_{l'}(f)$ and $l^o \overset{e}{\rightsquigarrow} l$. Thus, $m_{l^o}(f) < m_l(f)$. Since $m_l(\sigma \mid e) \le m_{l^o}(f)$ for any $\sigma \in \Xi(f, \mathcal{L})$, it holds that $\max_{\sigma \in \Xi(f,\mathcal{L})} m_l(\sigma \mid e) \le m_{l^o}(f)$. Let i be a player with $e(i) = e$ and $f_i = l^o$.

Ethnicity e is distinguished at l from the other relevant ethnicities by (4.3). Hence, when player i comes to l, it is possible for each player at l to take either a friendly or unfriendly action, independent of other conditions. To have a Nash equilibrium, the number of unfriendly players must be greater than $m_l(f) - m_{l^o}(f)$, since otherwise, player i could enjoy at least $m_l(f) - (m_l(f) - m_{l^o}(f)) + 1 = m_{l^o}(f) + 1 > m_{l^o}(f)$. Conversely, the smallest number of unfriendly players to have a Nash equilibrium is $m_l(f) - m_{l^o}(f) + 1$. Thus, we have a Nash equilibrium σ with $d_f(e \mid l) = m_l(f) - m_{l^o}(f) + 1 = m_l(f) - (\min_{l' \overset{e}{\rightsquigarrow} l} m_{l'}(f) - 1)$. □

The minimization operator in (4.4) may be necessary since e may be located in multiple locations, which may be possible by limited accessibility structure $\mathcal{L}$. When e is located only in one l', the minimization operator is unnecessary, i.e., $d_f(e \mid l) = m_l(f) - (m_{l'}(f) - 1)$. Let us look at Fig. 1.1; the players at l_1, l_2, l_3, l_4 can access all the locations. The players at l_1, l_2, l_3 can distinguish between e_1, e_2, e_3 and e_4. The no-confusion assumption (4.3) holds for e_2, e_3 at location l_1. Hence, when a player comes from l_2 to l_1 and from l_3 to l_1, respectively, he would meet the minimum discrimination

$$d_f(e_2 \mid l_1) = m_{l_1}(f) - m_{l_2}(f) + 1 \quad \text{and}$$
$$d_f(e_1 \mid l_2) = m_{l_2}(f) - m_{l_3}(f) + 1,$$

which are described as *AB* and *CD* in Fig. 1.1. If a player comes from l_1 to l_2 or l_3, or from l_2 to l_3, then $d_f(e_1 \mid l_2) = d_f(e_1 \mid l_3) = d_f(e_2 \mid l_3) = 0$ by Lemma 4.1(3).

Theorem 4.2 cannot be applied to the case where a player in l_1, l_2 or l_3 goes to l_4. The players in l_4 are not able to distinguish between e_1, e_2 and e_3. To express such indistinguishability, we make the following definition: let e be an ethnicity, and E, F sets of ethnicities. We define $e \sim_F E$ iff $e \sim_{e''} e'$ for all $e' \in E$ and $e'' \in F$. Note that $e \not\sim_F E$ is not the negation of $e \sim_F E$.

If two ethnicities are regarded as identical at l, then they are treated equally at l.

Lemma 4.3 (*Similar Players as Discriminated 1*). *Consider a location l and two ethnicities $e', e'' \in E^l(f, \mathcal{L})$ with $e' \sim_{E^l(f)} e''$. Then, $d_f(e' \mid l) = d_f(e'' \mid l)$.*

Proof. No players at l can distinguish between e' and e''. Hence, their responses to the presence of each are the same by (2.4). Hence $d_f(e' \mid l) = d_f(e'' \mid l)$. □

This is interpreted as a situation where each e in $E^l(f)$ has a large ethnic distance from two other groups e', e'' and cannot distinguish between e' and e''. This lemma can be regarded as describing the phenomenon (2) in Section 1.2. In the example of Fig. 1.1, ethnicities e_1, e_2, e_3 are indistinguishable at l_4. By Lemma 4.3, the players from l_1, l_2, l_3 face the same discrimination degree.

Lemma 4.3 itself sill allows some mixed cases: some ethnicity in $E^l(f)$ distinguish e' and e'' from themselves and some others may not. In such a case, the degree of discrimination may depend upon the ethnicity configuration at l and their observabilities about ethnicities. Here, we consider a clear-cut case.

Theorem 4.4 (*Similar Players as Discriminated 2*). *Let $l \in L_0$, $E \subseteq E^l_n(f, \mathcal{L})$ and $e \in E$. Assume that $e \sim_{E^l(f)} E$ but $e \not\sim_{E^l(f)} E^l(f, \mathcal{L}) \cup E^l(f) - E$, and that $m_{l'}(f) \le m_l(f)$ for some l' with $l' \overset{e}{\rightsquigarrow} l$. Then,*

$$d_f(e \mid l) = m_l(f) - \min_{l' \overset{e'}{\rightsquigarrow} l \text{ for some } e' \in E}(m_{l'}(f) - 1), \tag{4.5}$$

where l' is the controllable variable in the minimization operator.

Proof. By $e \sim_{E^l(f)} E$, all the ethnicities in E are treated equally by the players at l but are distinguished from the other ethnicities since $e \not\sim_{E^l(f)} E^l(f, \mathcal{L}) \cup E^l(f) - E$. By Lemma 4.3, all the ethnicities in E meet the same discrimination degree in each equilibrium. The minimum degree is determined by the smallest festival l' with $l' \overset{e'}{\rightsquigarrow} l$, $e' \in E$. It is enough to decrease the mood $m_l(f)$ to $m_{l'}(f)$ in order to kill the incentive for ethnicity e' to come l. This decrease is possible by the assumption $e \not\sim_{E^l(f)} E^l(f, \mathcal{L}) \cup E^l(f) - E$. We can have a Nash equilibrium to achieve $d_f(e \mid l)$ of (4.5). □

Let us return to the example of Fig. 1.1. Here, e_1, e_2 and e_3 are treated identically at l_4. Since e_3 forms the smallest festival at l_3, the minimum discrimination that any player from e_1, e_2 and e_3 faces at l_4 is the discrimination faced by e_3. Formally,

$$d_f(e_t \mid l_4) = m_{l_4}(f) - (\min(m_{l_1}(f), m_{l_2}(f), m_{l_3}(f)) - 1),$$

which is the length EF in Fig. 1.1.

Remark on the measure $d_f(\cdot \mid \cdot)$. According to (4.2), the value $d_f(e \mid l)$ may be supported by different Nash equilibria for different e and the same l or different l's. In fact, it holds that there is a Nash equilibrium $\sigma^o \in \Xi(f, \mathcal{L})$ such that for all $l \in L_0$ and all $e \in E^l(f, \mathcal{L})$,

$$m_l(\sigma \mid e) \leq m_l(\sigma^o \mid e) \quad \text{for any } \sigma \in \Xi(f, \mathcal{L}). \tag{4.6}$$

It follows from (4.6) that a common Nash equilibrium σ^o in $\Xi(f, \mathcal{L})$ supports any values of $d_f(\cdot \mid \cdot)$. This σ^o may not be unique, since only the number of discriminators are exactly determined.

5. Ethnic hierarchy, similarity, and distances

The phenomenon (2) mentioned in Section 1.2 was already discussed in Section 4. Here, we investigate phenomena (1), (3), and (4) in terms of the discrimination measure. Again, we assume $\Xi(f, \mathcal{L}) \neq \emptyset$.

First, we give the formal statement of the ethnic hierarchy result: the magnitude of discrimination faced by a group is reciprocally related to its position in the ethnic hierarchy.

Theorem 5.1 (*Ethnic Hierarchy*). *Let $l^1, l^2, \ldots, l^k \in L^0$, and for all $t, t' = 1, \ldots, k$, $m_{l^t}(f) > m_{l^{t'}}(f)$ if $l^t < l^{t'}$. Also, let $e^1, \ldots, e^k$ be ethnicities. We suppose that for $t, t' = 1, \ldots, k$,*

(a) $l^t \overset{e^t}{\rightsquigarrow} l^{t'}$ *if* $l^t \neq l^{t'}$;
(b) *the no-confusion condition (4.3) holds for e^t at $l^{t'}$ if $t \neq t'$.*

Then, $d_f(e^{t'} \mid l^t) = m_{l^t}(f) - m_{l^{t'}}(f) + 1$ for all $t, t' = 1, \ldots, k$ with $t < t'$.

Proof. By (a) and (b), the conditions of Theorem 4.2 hold. Hence, it follows that $d_f(e^{t'} \mid l^t) = m_{l^t}(f) - m_{l^{t'}}(f) + 1$. □

We can divide the assertion of Theorem 5.1 into two parts: When we focus on one festival location l^t, we have the hierarchy of discrimination degrees: a player from a smaller festival faces severer discrimination than one from a larger festival. That is,

$$\begin{aligned} d_f(e^1 \mid l^t) &= \cdots = d_f(e^{t-1} \mid l^t) \\ &= 0 < d_f(e^{t+1} \mid l^t) < \cdots < d_f(e^k \mid l^t). \end{aligned} \tag{5.1}$$

The first 0-discrimination part is obtained by Lemma 4.1(3). When we focus on one ethnicity, we find another hierarchy of discrimination degrees: A player of an ethnicity faces severer discrimination in a larger festival than in a smaller festival. That is,

$$\begin{aligned} &d_f(e^t \mid l^1) > \cdots > d_f(e^t \mid l^{t-1}) > 0 \\ &= d_f(e^t \mid l^{t+1}) = \cdots = d_f(e^t \mid l^k). \end{aligned} \tag{5.2}$$

Thus, the hierarchy result consists of (5.1) and (5.2).

This hierarchical structure is common to many multi-ethnic societies. According to Marger (1991), p. 135, American society is an example: it is divided into three social strata: the top stratum consists of white Protestants of various national origins; the middle mainly white Catholics and Jews, along with some Asians; and the lowest is made up of Blacks, Hispanics, Native Americans, and Asians. In this context, Theorem 5.1 has the claim: a higher stratum discriminates against a lower one; the middle stratum faces less discrimination than the lowest one; and the highest stratum faces no discrimination.

In Theorem 5.1, discrimination occurs in one direction: a person from a minority is discriminated against at a larger festival, while one from a larger to a smaller festival will face no discrimination. However, we can have mutual discrimination once we allow limited observability, which corresponds to (3) of Section 1.2.

Consider two locations l_1, l_2 with $e_1 \in E^{l_1}(f)$ and $e_2 \in E^{l_2}(f)$. Suppose $l_1 \overset{e_1}{\rightsquigarrow} l_2$, $l_2 \overset{e_2}{\rightsquigarrow} l_1$, and $m_{l_1}(f) \geq m_{l_2}(f)$. Then we have $d_f(e_2 \mid l_1) > 0$ by Lemma 4.1(2). Hence, we would like to give conditions for $d_f(e_1 \mid l_2) > 0$.

Theorem 5.2 (*Mutual Discrimination*). *Let l_1, l_2 be locations with $l_1 \overset{e_1}{\rightsquigarrow} l_2$ and $m_{l_1}(f) \geq m_{l_2}(f)$. Suppose that there are some l and e such that $l \overset{e}{\rightsquigarrow} l_2$, $e \sim_{E^{l_2}(f)} e_1$, and $m_{l_2}(f) \geq m_l(f)$. Then, $d_f(e_1 \mid l_2) > 0$.*

Proof. By Lemma 4.3, we have $d_f(e_1 \mid l_2) = d_f(e \mid l_2)$. By Lemma 4.1(2), we have $d_f(e \mid l_2) > 0$. □

Lemma 4.3, Theorems 4.4 and 5.2 are about discrimination faced by groups perceived as similar. The next result captures the other side of the issue. Suppose that a player of e comes to l from l', and he is facing some discrimination, i.e., $d_f(e \mid l) > 0$. There are already some players of E living in l whose ethnic distances to e are small. The people of E can distinguish e from themselves, which is condition (i) of Theorem 5.3. The other people, F, have larger distances to e, and they regard e as one type in E, which is condition (ii). Then, discriminators against e must belong to the similar ethnic group E.

Theorem 5.3 (*Similar Players as Discriminators*). *Let $e \in E_n^l(f, \mathcal{L})$ and $d_f(e \mid l) > 0$. Let $E^l(f)$ be partitioned into two nonempty sets E, F so that*

(i) *(Distinguishability by Similar): $e \nsim_E E$.*
(ii) *(Indistinguishability by Different): $e \sim_F E$.*

Then, in any Nash equilibrium supporting $d_f(e \mid l)$, any discriminator against e at l belongs to E.

Proof. When a player of e visits l, his presence is not observed at all by the players of F. Hence, no players in F do not respond to his presence. But $d_f(e \mid l) > 0$ means that some players become discriminators against e. They are not in F, so they must belong to E. □

The above phenomena can be observed in many instances, such as discrimination against new Indian immigrants by Indians living in the US; Indian Americans can distinguish new Indian immigrants from themselves, but many other Americans may not be able to. Other examples are: discrimination against new Chinese immigrants to Australia by Chinese Australians, and in New Zealand, discrimination by Maoris against South Islanders. See Marger (1991).

6. Prejudices and personal views

Limited accessibility $\mathcal{L} = \{L_i\}_{i \in N}$ has played a less explicit role in the previous section than limited observability $\{\sim_e\}_e$. However, it plays more roles in the study of prejudices. Prejudices are related more to *passive* experiences occurring when other people come to one's location, than to *active* ones occurring when one goes to other locations himself. Limited accessibility $\mathcal{L}$ may help classify individual experiences into active and passive experiences. This will add a large scope of research into prejudices to the approach of Kaneko and Matsui (1999). Here, we give a very brief discussion on a possible study of prejudices in the present framework.

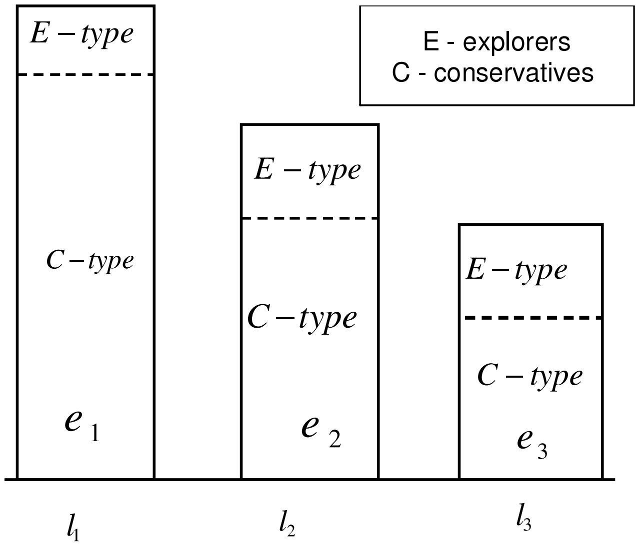

Fig. 6.1. Variety of players with different types.

Consider Fig. 6.1. In Kaneko and Matsui (1999), it is assumed that $L_i^E = \{l_1, l_2, l_3\}$ for all $i \in N$; every i of e_t $(t = 1, 2, 3)$ stays regularly at l_t and only sometimes goes to the other locations. Thus, all players at location l_t have active experiences uniformly. Consider this situation from the viewpoint of passive experiences. When a visiting player comes to l_1 from l_2 or l_3, some players may respond differently. There are four types of players at l_1 according to the responses to visiting players. They may develop simple explanatory views of the passive experiences induced by visiting players, or construct more complicated explanatory views (Kaneko and Matsui (1999), Section 6).

With limited accessibility $\{L_i\}_{i \in N}$, we have much greater variety of experiences and induced views. The polar opposite case to the above is: $\mathcal{L}^C = \{L_i^C\}_{i \in N}$ and $L_i^C = \{l_t\}$ for all players i with $e(i) = e_t$ and $t = 1, 2, 3$. That is, all are *conservative* in that players make no trial deviations and have no experiences other than their own festivals. This means that each location has no visitors. If a player at l_t is unaware of the other locations, it is a possible personal view that l_t is the sole world for him. If he knows the existence of the other locations, any personal view with an imaginary structure about the other world is compatible with his experience.

There are many intermediate cases between $\mathcal{L}^E = \{L_i^E\}_{i \in N}$ and $\mathcal{L}^C = \{L_i^C\}_{i \in N}$. In the following, we consider an intermediate one $\mathcal{L}^M = \{L_i^M\}_{i \in N}$, where at each location l_t, a majority of players are conservative (C-type, $L_i^M = L_i^C$), and the remaining minority consists of *explorers* (E-type, $L_i^M = L_i^E$) who occasionally go to other festivals.

Consider player i of C-type at l_2. His passive experiences are induced by visiting players from l_1 and l_3. In this case, $d_f(e_1 \mid l_2) = 0$ and $d_f(e_3 \mid l_2) > 0$, and player i has the following experiences:

(0) $[(l_2, 1), \{e_2\}, m_{l_2}(f)]$ — he stays at l_2;

(1) $[(l_2, 1), \{e_2, e_1\}, m_{l_2}(f) + 1]$ — a visitor comes from location l_1;

(2n) $[(l_2, 1), \{e_2, e_3\}, m_{l_2}(f) - d_f(e_3 \mid l_2) + 1]$ — a visitor comes from l_3, inducing discriminatory responses, but player i is not a discriminator;

(2d) $[(l_2, 1), \{e_2, e_3\}, m_0]$ — a visitor comes from l_3 and he behaves as a discriminator.

Since the number of players is large as stated by F1 in Section 2.1, we can assume that the additional $+1$ is ignored.

Consider case [(0), (1), (2n)]. Then, player i is indifferent about the presence of a visiting player of e_1, i.e., the utility value for (0) is (approximately) the same as that for (1). However, his utility value decreases a lot with the presence of a visiting player of e_3. Hence, he needs to explain this fact. One simple explanation for this pattern is the *naive hedonistic* view (Kaneko and Matsui, 1999): player i has a preference against e_3 but is indifferent about e_1. In the case of [(0), (1), (2d)], an explanation is similar, though his response to the presence of e_3 becomes unfriendly.

When player i is of E-type, he has the following two experiences in addition to (0)–(2) in the minimum discrimination equilibrium:

(3) $[(l_1, 1), \{e_1\}, m_{l_2}(f)]$ — player i goes to location l_1;

(4) $[(l_3, 1), \{e_3\}, m_{l_3}(f) + 1]$ — player i goes to location l_3.

Observe that the utility values in (3) and in (0) are the same. He may think that this same utility value is caused by the common component e_1 in $[(l_2, 1), \{e_2, e_1\}]$ and $[(l_1, 1), \{e_1\}]$, which is Occam's Razor: redundant explanatory variables should be eliminated. Thus, (0) and (3) may be explained by attributing the causes to the presence of e_1. Similarly, (2n) and (4) may be explained. Thus, player i does not need to extend his naive hedonistic explanation.

The above argument relies upon the minimum discrimination equilibrium supporting the measure $d_f(e \mid l)$. In a different equilibrium, an E-type player needs to develop a more sophisticated personal view than a C-type player.

The purpose of this section is simply to point out that our approach has much larger potential than in Kaneko and Matsui (1999) for the study of prejudices. A C-type player may learn from E-type players and change his view. Additionally, a player can learn from coethnic players with different views. If he has players of different ethnicities in his location, he may learn even more from them. In this respect, our approach also gives a framework for studying communication and resulting changes in attitudes.

7. Conclusions

This paper provided an analysis of discrimination and associated phenomena of segregation from the perspective of IGT. Social interaction was modeled as the festival game of Kaneko and Matsui (1999), which was extended in this paper by introducing additional constraints on the observability of ethnic identities and on locations accessible by the players. These constraints reveal a greater variety of stable segregation patterns and discriminatory behavior than in Kaneko and Matsui (1999). The basic idea and perspective come from the recent development of IGT in Kaneko and Kline (2008a,b). This paper shows the applicability of IGT to general societal phenomena.

Specifically, we provided a characterization of the Nash equilibrium set of the festival game with limited observability and accessibility. We characterized the set of Nash equilibria relative to a given location configuration. This characterization allowed us to introduce a measure of discrimination, interpreted as the minimum degree of discrimination needed to sustain a location configuration as equilibrium. We then used the measure to investigate discrimination in an ethnic hierarchy; discrimination faced by similar ethnicities; mutual discrimination; and discrimination by similar ethnicities. We can expect more phenomena to be studied, and an exploration of them will be a part of future research.

We have briefly pointed out in Section 6 that the additional structures imposed on the festival game have great potential for the study of prejudices, too. While a detailed study from the perspective of IGT will be conducted in a future paper, we have taken an initial step in this direction.

Conceptually, an analysis of the emergence of prejudices is important for the continuation of our research on discrimination. In this paper, we have only considered "individualistic" discrimination in the sense that when a player of a different ethnicity visits a festival, the players have discriminatory responses, but there is no "institutional" discrimination in the sense that they do not organize political campaigns or develop institutional arrangements to intensify discrimination against some ethnic groups. Institutional discrimination may occur when prejudices associated with individualistic discrimination are developed and a trigger is pulled. It will be possible to consider this when individualistic discrimination and the emergence of prejudices are fully analyzed.

In conclusion, the festival game captures social interactions in a highly abstract manner, concentrating on discriminatory behaviors that arise as a part of group formation and eliminating other socio–economic components. Due to this abstraction, we are able to study various segregation patterns and discriminatory behaviors. These are suggestive for empirical studies of intergroup relations. Nevertheless, we admit that our theory cannot directly be connected to empirical studies; both because it is a highly simplified and focused theory on discrimination and prejudices and because in reality, institutional backgrounds such as colonialism is significant to be ignored. Our theory is suitable to a heuristic use for the study of discrimination and prejudices.

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